When Balassa-Samuelson comes to Maastricht: Debt-sustainability, growth and transition

Trond-Arne Borgersen
Roswitha M. King

Høgskolen i Østfold
Arbeidsrapport 2011:3

Online-versjon (pdf)

Utgivelsessted: Halden

Høgskolen i Østfold har en godkjenningsordning for publikasjoner som skal gis
ut i Høgskolens Rapport- og Arbeidsrapportserier.

Høgskolen i Østfold. Arbeidsrapport 2011:3

ISBN: 978-82-7825-343-4
ISSN: 1503-6677

When Balassa-Samuelson comes to Maastricht:
Debt-sustainability, Growth and Transition

Trond-Arne Borgersen
Østfold University College
Department of Economics
Remmen
N- 1757 Halden
e-mail: trond.a.borgersen@hiof.no

Roswitha M. King
Østfold University College
Department of Economics
Remmen
N- 1757 Halden
e-mail: roswitha.m.king@hiof.no

Abstract:

A dynamic stability approach is applied to show how the debt-servicing ability of transition/developing economies exceeds that of mature market economies. At the core of this debt-servicing advantage lies not only relatively higher growth but more importantly the productivity- and efficiency gains released by the re-allocation and restructuring activities originating in the developing economies' unbalanced sectoral growth. First, the dynamic stability approach is extended to allow for two sectors of production, allowing both growth and sectoral composition to matter for stability and the debt-to-GDP ratio. Second, sector sizes are endogenized to depend on the productivity growth difference. The model shows how the equilibrium debt-to-GDP ratio depends on both an economy's existing industrial structure as well as on its structural flexibility and how it varies along a country's transition trajectory. All things considered, how much debt relative to GDP a country can (or should) sustain is highly context specific and depends on the prevailing economic structure, composition of growth, structural flexibility, and the prevailing incentives for restructuring. Hence this paper delivers a cautionary note toward the one-size-fits-all approach to debt sustainability.

JEL Classifications:

Keywords: Debt sustainability, growth, transition, stability, economic structure

1. Introduction

The economic slowdown in European economies in the early 2000s shortly after the introduction of the Stability and Growth Pact (SGP) already exposed weaknesses in a scheme designed to support (if not ensure) fiscal stability in the Economic and Monetary Union (EMU). The magnitude of the current pressures on the mechanism, originating from costly measures to avert a global financial crash, is of a scale that it was hardly designed to sustain. The original agreement on the SGP came after much politicking by Germany to ensconce the prohibition against 'excessive deficits' from the Maastricht Treaty, at its centre. One of the drivers of this concern, particularly on the part of the European Central Bank (ECB) is that excessive deficits would eventually be monetized and thus could endanger price stability in the EMU. Currently of more immediate concern is the sustainability of debt, in the sense that the increasing burden of debt servicing may inflict serious injury to an economy and may push the economy beyond a 'point of no return', from which it has to be rescued by others at great cost – not to mention the additional moral hazard problems that this may raise. How much debt can a country carry? How much debt is too much? And is there a universal indicator? Having arrived at a point in European Union (EU) history where countries at the 'inner core' of the union, i.e. EMU member states, may have stepped beyond their point of no return[1], and new EU member states in Eastern Europe stand by and watch with astonishment, debt sustainability analysis is not only of theoretical interest.

This paper seeks answers to the following pair of questions: Do mature market economies, such as the original EU member states, and transition economies, such as the economies of Central and Eastern Europe, differ systematically in their capacity to sustain debt levels relative to GDP? And if so - why?

[1] As amply evidenced by the case of Greece and Ireland.

A second related question targets only transition economies. Can we stratify transition economies according to differential debt sustainability thresholds? And if so – how?

Our point of departure is the Domar (1944) approach to dynamic stability assessment of the public debt/GDP ratio. We extend the Domar model by introducing two sectors of production – a sector that produces tradables and one that produces non-tradables – along the lines of the Scandinavian Model of Inflation (SMI) (Lindbeck, 1979) – one of its components being the more widely known Balassa-Samuelson model. The two sectors of production allow us to relate assessments of debt stability to sectoral differences in productivity growth and to structural inflation. As elaborated below, some part of the productivity growth differences induces structural shifts, while another part does not. To distinguish between the two the reasoning of the Scandinavian Model of Inflation is extended by endogenizing sector sizes and introducing the metric of structural flexibility. This distinction allows us to sort transition economies along two profiles: Those who can *and* should have higher debt-to-GDP ratios (relative to mature market economies) and those who can *but should not.* These model based normative assessments are related to the degree of structural flexibility in transition economies.

The Maastricht criteria were designed to promote (if not insure) responsible governance of economic affairs and individual countries' economic convergence to a point where a single monetary policy for all of euroland is workable. The criteria include price stability, exchange rate stability, restrictions on public debt and deficits and long-run interest rate variability (European Commission 2006). However, it seems as though it is the criteria on debt and deficits that have most frequently come under public scrutiny. For instance, Buiter (2006) claims the Maastricht deficit and debt criteria to be arbitrary and neither necessary nor sufficient for national financial sustainability. De Grauwe (2009) declares the Maastricht criteria in general and the debt-deficit criteria in particular to be political instruments that are not economically vital measures at all. And presently, what distinguishes the debt and deficit criteria, is that they are NOT satisfied by those who pledged that they would.

In the literature on sustainability of government deficit and/or debt the contributions are often grouped around two methodological approaches: the *'solvency criterion'*, which focuses on deficits rather than debt, and is anchored on an inter-temporal budget constraint (see for example Haliassos and Tobin 1990, Hamilton and Flavin 1986, Wilcox 1989, Persson 1985) and the *'dynamic stability approach'* attributed to Domar (1944), which examines dynamic stability of the public debt/GDP ratio around a steady state (see for example Tobin 1986, Heise 2002, Pasinetti 1998, Blanchard et al. 1990, Spaventa 1987, Masson 1985).

Although ubiquitous in the literature and cherished for its elegance, the solvency criterion has been found rather unsatisfactory on operational grounds, and among some critics also on theoretical grounds (see for instance Corsetti and Roubini, 1991).

The dynamic stability approach, while not without critics, offers the advantage that sustainability can be defined in terms of meaningful macroeconomic parameters. It is this feature that makes the approach desirable for the inquiry of this paper.[2]

The results of this paper are in contrast to Reinhart and Rogoff (2010) who, in their empirical study of a large set of countries, arrive at the conclusion that debt/GDP thresholds are similar

---

[2] This approach, which is anchored on the debt/GDP ratio, is also attractive on operational/empirical grounds. Recent history of public finance in the EMU has shown increasing evidence of "off-budget expenditures" (Gros 2003). This has led to substantial discrepancies between the stock of public debt and the accumulation of budget deficits over time – with the stock of deficits often far exceeding the accumulated budget deficits. As a consequence we have witnessed a number of substantial "stock-flow" adjustments – of a magnitude that makes it difficult to assess whether or not a country is actually satisfying the Maastricht criteria on debt and deficits.

for mature and transition[3] economies. This paper, however, finds significant differences in debt sustainability thresholds of these two types of economies. It should be noted that while at the core of both papers lies the relation between economic growth and public debt, the implied causal ordering is reversed. While Reinhart and Rogoff (2010) essentially look at what debt does to growth sustainability this paper looks at what growth does to debt sustainability.

A recent empirical paper by Becker et al (2010) broadly supports our analytical results. Their paper offers empirical estimations and scenario forecasts of debt sustainability for 38 countries, consisting of both developed markets and emerging markets – together accounting for about 85% of world GDP. One of their central and striking findings is that the emerging markets are in much better shape than the developed markets regarding debt sustainability. Becker (2011) focuses on Latin American economies. The central message is that it is not only the amount but also the structure of debt that matters. While Becker essentially looks as what the *structure of debt* does to its sustainability, this paper looks at what *the structure of GDP* does to the sustainability of debt.

This paper provides theory that allows for a finer stratification of debt sustainability thresholds and a more differentiated understanding of its causes. Several key messages emerge from the analytics: Is it is not just growth that matters but the composition of growth is essential for assessing debt sustainability. Transition economies can sustain higher debt-to-GDP ratios compared to mature market economies. Among the transition economies those with higher structural flexibility can sustain higher debt-to-GDP ratios than those with lower flexibility. Furthermore a country's prevailing industrial structure matters.

[3] More generally, emerging economies.

The paper is organized as follows. In section two we present the basic elements of the Domar (1944) model. In addition the two sector structure is introduced. In the third section we compare stability in a mature economy, a transition economy with fixed sector sizes and in a transition economy where sector sizes are endogenously related to the productivity growth differential. The section also introduces a metric for sorting transition economies. The last part concludes.

2. Growth versus transition in a two sector Domar model

In this section, we develop a stylized model of debt-stability that incorporates two sectors of production. We apply a definition of transition which enables the model to distinguish between mature and transition economies. Being associated with structural shifts, transition is related to the incentives for such shifts: differences in productivity growth between sectors and the resulting structural inflation. The model links structural inflation with debt sustainability. In particular it highlights the implications of structural inflation on the conditions for debt-stability. First, the implications of structural inflation when sector sizes are given are derived. Second, we allow for changes in relative sector sizes by making them endogenous to the model.

A. Aggregate production and public debt

Along the lines of Domar (1944) aggregate GDP, *Y,* is assumed to grow at a fixed rate *n,*

(1) $$Y_t = Y_0 e^{nt} .$$

The public sector's budget constraint states that government expenditures *G* and interest payments on existing debt *R* must be financed by either taxes *T* or additional borrowing *D*

(2) $$T_t + \dot{D}_t = G_t + R_t$$

The public sector is assumed to borrow at fixed rate *a (a>0)* of GDP each period $\dot{D}_t = aY_t$ while the interest payments on existing debt equals $R_t = D_t i$. The interest rate is given by *i (i>0)*. The government taxes both the real side and the financial side of the economy with the rate τ_0>0

(3) $$T_t = \tau_0 \left(Y_t + R_t \right).$$

B. Growth versus Transition

We now allow for two sectors of production, a tradable sector and a non-tradable sector. We define aggregate growth as a weighted average of productivity growth in the two sectors. Relative sector sizes function as weights.

DEFINITION 1: *Aggregate growth*

$$n \equiv \alpha \dot{q}_T + (1 - \alpha) \dot{q}_{NT} \qquad \alpha \in (0,1)$$

where productivity growth in the tradable sector is given as $\dot{q}_T \in (0,1)$ and productivity growth in the non-tradable sector as $\dot{q}_{NT} \in (0,1)$. The relative size of the tradable sector equals α and the size of the non-tradable sector is denoted by (1-α).

We introduce the following definitions:

DEFINITION 2: *A mature economy:*

$$\dot{q}_T = \dot{q}_{NT} \quad \Rightarrow \quad n = \dot{q}_{NT} = \dot{q}_T$$

The definition of a mature economy states that it is characterized by equal productivity growth in all sectors of production. Growth is thus equal to productivity growth in the tradable (and in the non-tradable) sector.

By contrast, a transition economy is described by unequal productivity growth across sectors, i.e. $\dot{q}_T \neq \dot{q}_{NT}$. Growth is unbalanced. Moreover we relate transition to the assumption of the Balassa-Samuelson model that productivity growth in the tradable sector exceeds that of the non-tradable sector.

DEFINITION 3: *A transition economy:*

$$\dot{q}_T > \dot{q}_{NT}$$

It is the unbalanced growth that acts as an *incentive for structural shifts* in transition economies. Thus we implicitly define a transition economy as one in which incentives for structural shifts exist.

To incorporate the Balassa-Samuelson effect into the Domar model we express aggregate growth as the difference between the productivity growth in the tradable sector and structural inflation $\dot{P}_{STR}$.[4]

(4) $$n = \dot{q}_T - (1-\alpha)\left[\dot{q}_T - \dot{q}_{NT}\right] = \dot{q}_T - \dot{P}_{STR},$$

which follows directly from Definition 1 after some algebraic manipulation. This allows us to characterize transition economies according to the prevailing degree of structural inflation.

So, the basic distinction between a mature economy and a transition economy is that aggregate growth in the latter is split in two components; a *general increase* in value added and the potential change in the *distribution* of value added between sectors following differences in productivity growth.

[4] Where $(1-\alpha)\left[\dot{q}_T - \dot{q}_{NT}\right] \equiv \dot{P}_{STR}$ represents the Balassa-Samuelson effect.

C. *Discussion of our setup:*

Two features of our modeling framework need further discussion: The reasoning underlying the Balassa-Samuelson effect, and the conditions for changes in the distribution of value added.

The first feature can - along the lines of Lindbeck (1979) - be related to the Scandinavian Model of Inflation. We assume that an economy consists of two sectors of production: a tradable and a non-tradable sector. In both sectors of production capital K and labor L are used as inputs. Labour is homogenous and factor income shares are fixed. While the share of output (Y_T) that is spent on wages (w_T) in the tradable sector is $\frac{w_T L_T}{P_T Y_T} = \sigma_T$ the share of output in the non-tradable sector that falls into the hands of labour equals $\frac{w_{NT} L_{NT}}{P_{NT} Y_{NT}} = \sigma_{NT}$, where P_T and P_{NT} denotes prices of tradables and non-tradables respectively. There is a fixed ratio between wages in the two sectors, equalising wage growth $(\dot{w})$. When it comes to pricing, the two sectors differ: purchasing power parity is (variables expressed as rates of change) assumed to govern pricing of tradable goods ($\dot{p}_T = \dot{p}_w + \dot{e}$), were $\dot{p}_w$ denotes world price and $\dot{e}$ denotes the relevant exchange rate. There is mark-up pricing in the non-tradable sector, as prices on non-tradables (p_{NT}) are set in relation to unit labour cost $(\dot{p}_{NT} = \dot{w} + \dot{q}_{NT})$. To ensure that factor income shares are fixed, wage growth is determined in the tradable sector $(\dot{w} = \dot{p}_T + \dot{q}_T)$. Assuming a price index that is a weighted sum of prices in the two sectors - using relative sector sizes as weights – overall inflation equals $\dot{P} = \alpha\dot{p}_T + (1-\alpha)\dot{p}_{NT}$. Substituting for price growth on tradables and non-tradables, domestic inflation can be expressed as $\dot{P} = \dot{p}_W + \dot{e} + (1-\alpha)\left[\dot{q}_T - \dot{q}_{NT}\right]$. In a small open economy inflation now has two components, imported inflation $\dot{p}_W + \dot{e}$, and a domestic component. The latter - which is the Balassa-Samuelson effect - is denoted structural inflation.

(5) $$\dot{P}_{STR} = (1-\alpha)\left[\dot{q}_T - \dot{q}_{NT}\right]$$

In our expression (4) for aggregate growth in transition economies the B-S effect enters with a negative sign. This is due to that sector sizes are fixed, even though productivity growth differs between sectors and thus incentives for structural shifts exist. By the above reasoning this implies that the non-tradable sector is exposed to the same wage growth rate as the tradable sector, even if productivity growth in the non-tradable sector is lower. The fixed factor income share[5] for labour channels a part of the productivity-generated increase in value added to subsidise labour in the less efficient sector of production – pushing wages above the marginal product of labour. This puts a drag on the economy that, overall, is welfare reducing. A mature economy is not exposed to this kind of drag by construction of the model. Starting out with a model where incentives for structural shifts are present but are wasted since no shifts actually occur, mimics our understanding of a suppressed feature in the literature on economic transition: Transition is not a trivial process and can be hampered by institutional features.

In its pure form the Scandinavian Model of Inflation illustrates a perfectly inflexible economy, characterised by severe rigidities hampering transition. The degree of structural inflation can now be interpreted as the extent of subsidy in the economy.

[5] The model assumes that wage growth is equalized between sectors. While this may appear to be a rather strong assumption, there is empirical evidence in support of it. For example, Égert et al (2002) analyse a number of transition economies. They find in most of the countries that (i) real wage growth in the tradable sector is connected to productivity growth, and (ii) that wage increases tend to equalize between the tradable and the non-tradable sectors. Hence, both provide support for the underlying model assumptions.

When analysing transition, however, structural shifts must be accounted for and the reasoning of the Scandinavian Model of Inflation must thus be extended. The question then arises: what are the conditions for structural shifts? Stated differently, what are the conditions for changes in the distribution of value added? Following Borgersen and King (2009) we endogenize sector sizes - defining them as increasing functions of sector-favouring changes in productivity growth differentials. This set-up allows us to distinguish between a part of the productivity growth differential that results in structural shifts and a part which does not. Hence, while a part of the productivity growth differential still is *wasted,* structural shifts and the economy's *ability* to take advantage of the incentives for such shifts are now included. That is, structural flexibility is incorporated. This framework allows us to distinguish between economies where the incentives for transition are utilised and economies where they are not. By endogenizing sector sizes, and making the size of the tradable sector an increasing function, *g*, of the productivity growth differential, $\alpha = g(\dot{q}_T - \dot{q}_{NT})$, the condition for when a higher productivity growth differential stimulates structural inflation is related to the sector size elasticity. The elasticity measures the extent to which sector sizes are affected by the productivity growth differential, and is an indicator of structural flexibility. The degree of structural flexibility is shown to matter for the debt-servicing ability of a transition economy. Motivating the two sector Domar model by the reasoning of the SMI provides the means for distinguishing between *rigid* transition economies with substantial subsidising of less efficient factors of production which *could, but should not* have higher debt-to GDP ratios and *flexible* transition economies which *could and should* carry higher debt-to GDP ratios.

In section 3 the stability conditions and sustainable debt-to-GDP ratios are discussed. First we derive the debt-to-GDP ratio in a mature economy. Second, the debt-to GDP ratio is derived in a transition economy with fixed sector sizes. Third, we allow for endogenous sector sizes relating debt-servicing ability to growth, flexibility, structural status quo, and cross-sectoral subsidies.

3. Debt stability in mature and transition economies

Within the basic structure above a comparison between debt-stability in mature and transition economies can be drawn.

A. A mature economy

In the Domar-type model described above, solutions can be stable or not (Muckl, 1983). In the following we only consider the stable solutions. When using the definition of a mature economy, $n = \dot{q}_T$, stability is constrained by tradable sector productivity growth exceeding the net interest rate

Assumption 1: $\dot{q}_T^{MAT} > (1-\tau_0)i \equiv Q^{MAT}$

In a stable equilibrium the debt-to-GDP ratio equals

$$\lim_{t\to\infty}\left(\frac{D_t}{Y_t}\right) = \frac{a}{\dot{q}_T{}^{MAT} - (1-\tau_0)i} \tag{6}$$

Conventionally the debt-to-GDP ratio increases in both the borrowing rate and the interest rate, while it decreases in the tradable sector productivity growth rate and the tax rate.

In the following we will, for notational simplicity, omit the superscripts 'TR' and 'MAT' whenever it is clear from the context that we are talking about a transition economy or a mature economy respectively.

B. *A transition economy with fixed sector sizes*

When using expression (4) for growth in a *transition* economy, and assuming equal tax rates and interest rates between mature and transition economies, the stability condition equals

Assumption 2: $\dot{q}_T^{TR} > (1-\tau_0)i + \dot{P}_{STR} \equiv Q^{TR}$

In the following we will refer to Q^{MAT} and Q^{TR} as the stability thresholds of tradable sector productivity growth that ensure stability in a mature and in a transition economy respectively. The assumption of equality of interest rates and tax rates between mature and transition economy serves to isolate the effect of structural origins.

Hence, when comparing the conditions for stability in a transition economy with that of a mature economy we may state the following:

Lemma 1: *Under conditions of equality of interest rates and tax rates the thresholds for tradable sector productivity growth that ensure stability in a mature and in a transition economy respectively are related as follows:* $Q^{TR} > Q^{MAT}$.

This follows directly from assumption 1 and 2 under observance of definition 1 and 3, and expression (5).

Let us for instance assume an interest rate of 5 percent, a tax rate of 40 percent and a structural inflation rate of 5 percent. For ensuring stability the tradable sector productivity growth in a mature economy must exceed 3 percent ($\dot{q}_T^{MAT} > 0{,}03$), while it must exceed 8 percent $\left(\dot{q}_T^{TR} > 0{,}08\right)$ in a transition economy. Hence, when comparing the conditions for stability between mature and transition economies some *catching-up* in tradable sector productivity growth is necessary for stability in transition economies.

By assumption 2 the stability threshold of tradable sector productivity growth rate necessary for stability in transition economies is positively related to the degree of structural inflation. Analogously to expression (6) above, in a transition economy the long run debt-to-GDP ratio equals

$$(7) \qquad \lim_{t\to\infty}\left(\frac{D_t}{Y_t}\right) = \frac{a}{\dot{q}_T{}^{TR} - \dot{P}_{STR} - (1-\tau_0)i}$$

Again, the ratio increases in the borrowing rate and in the interest rate while it decreases in both the tradable sector productivity growth rate and the tax rate. Contrary to a mature economy, the structural inflation component, $\dot{P}_{STR}$, also matters for the debt- to- GDP ratio. As explained by (5) structural inflation is derived from the productivity growth differential and the size of the non-tradable sector, and in the model it measures the extent of subsidizing that occurs in the transition economy. At the same time does transition economies' tradable sector productivity growth exceed that of its non-tradable sector. The difference is an indicator of what the economy can spend on subsidies to the less efficient non-tradable sector and matters for debt sustainability. When sector sizes are fixed structural inflation measures the subsidizing element of the economy.

When comparing transition economies to mature economies, and adhering to assumptions 1 and 2, the following arguments can be derived:

Lemma 2: *Under conditions of equality of interest rates, tax rates and borrowing rates the sustainable debt-to-GDP ratio is equal in a mature and in a transition economy if the difference between the respective tradable sector productivity growth rates equals structural inflation in the transition economy,* $\dot{q}_T^{TR} - \dot{q}_T^{MAT} = \dot{P}_{STR}$.

Proof: It follows directly from (6) and (7) that if $\dot{q}_T^{TR} - \dot{q}_T^{MAT} = \dot{P}_{STR}$

$$\lim_{t\to\infty}\left(\frac{D_t}{Y_t}\right)\Bigg|_{\dot{q}_T^{MAT}=\dot{q}_T^{TR}-\dot{P}_{STR}}^{MAT} = \lim_{t\to\infty}\left(\frac{D_t}{Y_t}\right)^{TR}$$

So the sustainable debt-to-GDP ratios of a mature and a transition economy coincide in the case that higher productivity growth in a transition economy relative to a mature economy is equal in amount to structural inflation of the transition economy. When all the catching up in tradable sector productivity growth is spent on subsidies the debt-to-GDP ratio in mature and transition economies are equal. This as transition economies, despite catching up in tradable sector productivity growth, cannot expect additional increases in future growth. The case of Lemma 2 is, however, rather special and we expect that the more usual case is that where $\dot{q}_T^{TR} - \dot{q}_T^{MAT} \neq \dot{P}_{STR}$. This case is addressed in the following:

Lemma 3: *Under conditions of equality of interest rates, tax rates and borrowing rates: if the difference between the respective tradable sector productivity growth rates is less than structural inflation in the transition economy,* i.e. if $0 < \dot{q}_T^{TR} - \dot{q}_T^{MAT} < \dot{P}_{STR}$,

the debt-to GDP ratio in a transition economy exceeds that of a mature economy. If the difference between the respective tradable sector productivity growth rates exceeds structural inflation in the transition economy, $\dot{q}_T^{TR} - \dot{q}_T^{MAT} > \dot{P}_{STR}$, *the debt-to-GDP ratio in a transition economy is lower than in a mature economy.*

It follows again directly from (6) and (7) that if $0 < \dot{q}_T^{TR} - \dot{q}_T^{MAT} < \dot{P}_{STR}$

$$\lim_{t\to\infty}\left(\frac{D_t}{Y_t}\right)\Bigg|_{\dot{q}_T^{MAT}<q_T^{TR}-\dot{P}_{STR}}^{MAT} < \lim_{t\to\infty}\left(\frac{D_t}{Y_t}\right)^{TR}$$

Likewise, if $\dot{q}_T^{TR} - \dot{q}_T^{MAT} > \dot{P}_{STR}$

$$\lim_{t\to\infty}\left(\frac{D_t}{Y_t}\right)\Bigg|_{\dot{q}_T^{MAT} > q_T^{TR} - \dot{P}_{STR}}^{MAT} > \lim_{t\to\infty}\left(\frac{D_t}{Y_t}\right)^{TR}$$

Proof: See appendix for a formal proof.

When sector sizes are fixed, no structural shifts accompany the productivity growth differential. Motivating structural inflation by the reasoning of the Scandinavian Model of inflation a higher debt- to-GDP ratio in transition economies can be argued necessary for subsidizing labor and maintaining fixed factor income shares in the less efficient sector of production. The structural inflation component equals the subsidy necessary to maintain status quo. If structural inflation exceeds the difference between tradable sector productivity growth rates in a transition and in a mature economy, the transition economy is in *need* of a higher debt-to-GDP ratio in order to finance the redistribution of income. On the other hand, if structural inflation is lower than the difference between the respective productivity growth rates, the economy can get by with a lower debt-to GDP ratio as the need to subsidize labor in the less efficient sector of production is smaller.

When in transition economies the extent of subsidizing exceeds its catching up in tradable sector productivity (to mature economies) the debt-to-GDP ratio is lower than that of mature economies, simply because one spends more on subsidizing the less efficient factors of production than the catching up in productivity growth allows for. When the extent of subsidizing is smaller than the catching up in tradable sector productivity growth, the sustainable debt-to-GDP ratio is higher, as some of the productivity gain can be expected to result in higher growth, even for given sector sizes.

Lemma 2 and 3 already point to a relation between structural inflation and the sustainable debt-to-GDP ratio. When comparing among transition economies it turns out that the debt-to-

GDP ratio increases when structural inflation increases, so that transition economies with higher structural inflation can sustain a higher debt-to-GDP ratio relative to economies with comparatively lower structural inflation - a relation made more explicit in the following.

Lemma 4: *The debt-to-GDP ratio is positively related to structural inflation.*

This can be seen by taking the derivative of expression (7) with regard to structural inflation:

$$\frac{d\left(\lim_{t\to\infty}\frac{D_t}{Y_t}\right)}{d\dot{P}_{STR}}=\left(\frac{a}{\left(\dot{q}_T-\dot{P}_{STR}-(1-\tau_0)i\right)^2}\right)>0\ .$$

The higher the structural inflation the bigger is the share of the value added that is spent on subsidizing labor in the less efficient sector of production. By the definition of 'structural inflation' given in (5) the extent of the need for such subsidies depends on two factors: (i) by how much the productivity growth in the non-tradable sector lags behind that of the tradable sector, but also on (ii) the relative share of the non-tradables in producing value added in the economy. Countries which spend a high share of value added on subsidizing the less efficient sector of production have a relatively greater need for debt financing, and hence a need for a relatively higher debt-to-GDP ratio.

Lemma 2 and 3 make statements regarding debt- to-GDP ratios in mature and transition economies at the stability margins. A relatively higher tradable sector productivity growth rate in a transition economy naturally brings about an ability to service a higher debt-to-GDP ratio in long run equilibrium if necessary. Whether it is necessary or not, depends on the degree of structural inflation and the extent of redistribution.

As Lemma 4 relates sustainable debt to structural inflation, it should not be mixed with arguments regarding countries ability to 'inflate away debt', as structural inflation relates to changes in relative prices, and not changes in the general price level of an economy.

The lemmas are also derived under the condition of equal borrowing rates. In financial markets however transition economies face higher interest rates. Allowing for an interest rate spread between mature and transition economies would (invalidating the premises of our lemmas) show how the debt-to-GDP ratio in transition economies could be lower than in mature economies *ceteris paribus*[6].

Even so, leaving aside margin assessments and the implication of interest rate spreads the model carries with it a more interesting feature regarding debt sustainability. To isolate this feature from effects coming simply from differences in productivity growth, we now assume that the productivity growth rates in the tradable sector are equal across transition and mature economies and that stability conditions are fulfilled for both of them. In this case assessment of debt- to-GDP ratios brings another dimension into debt servicing ability. The following proposition highlights the inherent positive implication for debt servicing ability in transition economies originating in the incentives for structural shifts.

[6] Mitigating this caveat is the fact that during the past decade transition economies have borrowed heavily in foreign currencies – often at interest rates below their domestic inflation rates.

Proposition 1: *Suppose a common interest rate, tax rate and borrowing rate, as well as a common tradable sector productivity growth rate in mature and transition economies,* $\bar{q}_T$, *which satisfies the stability condition of Assumption 2.*

$$\text{Then } \lim_{t\to\infty}\left(\frac{D_t}{Y_t}\right)^{TR} > \lim_{t\to\infty}\left(\frac{D_t}{Y_t}\right)^{MAT}$$

Proof: See appendix.

In a transition economy where growth is unbalanced and the economy is exposed to incentives for improving the allocation of resources, the debt servicing ability is greater than in a mature economy. This allows a higher debt-to GDP ratio in transition economies although tradable sector productivity growth is equal between the two. As sector sizes are fixed and the incentives for improving the allocation of resources are not taken advantage of, transition economies can be said to be in need of a higher debt-to-GDP ratio in order to finance the subsidizing of labor in the less efficient sector of production.

C. *A transition economy with changes in sector sizes*

So far debt servicing ability in transition economies has been assessed with fixed sector sizes. The higher debt-to-GDP ratio in transition economies derived above can be claimed necessary due to the economies' structural inflexibility and represents the cost of maintaining status quo. Stated differently: Transition economies carry higher debt-to-GDP ratios because they need to do so.

However, the productivity growth differential also carries with it a positive implication for the debt servicing ability of transition economies. Transition is related to changes in the distribution of value added, and following Blanchard (1997) essentially in favor of the non-tradable sector[7]. Structural shifts can be argued on the basis of productivity growth differentials and structural inflation. As shown by Borgersen and King (2009) the presence of incentives for structural shifts alone will not inevitably result in structural shifts. Furthermore, to make precise assessments on how the debt servicing ability is affected by structural shifts it is necessary to endogenize the relationship between sectoral size and the productivity growth differential, as already indicated in section 2 above. To give this idea some purchase we give this relation the following form:

(8a) $$\alpha = \left(\dot{q}_T - \dot{q}_{NT}\right)^b ,$$

which implies

(8b) $$(1-\alpha) = 1 - \left(\dot{q}_T - \dot{q}_{NT}\right)^b ,$$

where $b > 0$ equals the sector size elasticity, measuring how much a one percent change in the productivity growth differential changes the tradable sector's share of value added[8] . The relative size of the tradable sector is increasing in the productivity growth differential.

It is decreasing[9] in the sector size elasticity, which implies that the relative size of the non-tradable sector is increasing in sector size elasticity.

The use of the above described structural flexibility measure allows us to model structural shifts – which lie at the heart of the transition process. This not only allows us to sharpen our comparative analysis of debt stability/debt service ability of a generic transition economy

---

[7] In the modeling framework adopted in this paper this means, since $\dot{q}_T > \dot{q}_{NT}$ by assumption, that transition implies a shrinking of the productivity growth difference $\left(\dot{q}_T - \dot{q}_{NT}\right)$.

[8] As the size of the tradable sector increases in the productivity growth differential, the size of the non-tradable sector decreases in the same differential and vice versa.

[9] This owes to the assumptions that $\dot{q}_T$, $\dot{q}_{NT} \in (0,1)$ and $\dot{q}_T > \dot{q}_{NT}$ and, by implication, $\left(\dot{q}_T - \dot{q}_{NT}\right) \in (0,1)$, together with the properties of natural log.

versus a generic mature economy, but, moreover, it lets us differentiate between types of transition economies. We will deal with both of these perspectives in turn – beginning with the latter one.

Differentiating among transition economies

Defining structural flexibility in terms of sector size elasticity and defining transition as an increase in an economy's share of the non-tradable sector allows to relate structural flexibility to transition.

Lemma 5: *Structural flexibility is a transition driver and sectoral imbalances in productivity growth act as a transition retardant.*

Proof in Appendix.

The message of Lemma 5 is that, as a transition economy becomes more flexible, the transition process accelerates. This, by our working definition of transition, means that the speed at which the non-tradable sector increases relative to the tradable sector picks up. If the relative increase in the non-tradable sector is accompanied and/or caused by a relative increase of that sector's productivity growth (and thus, other things constant, the sectoral productivity growth difference declines) that too accelerates transition. In that case increased flexibility and the economy's shift toward an increasingly productive non-tradables sector support each other in driving transition. Conversely, transition economies lacking these properties tend to stagnate or move backwards on the transition trajectory.

When using expression (4) and substituting for α from (8) the aggregate growth rate of a transition economy can be expressed as

(9) $$n = \dot{q}_T - \left[1-(\dot{q}_T - \dot{q}_{NT})^b\right]\left[\dot{q}_T - \dot{q}_{NT}\right] = \dot{q}_T - \left[\dot{q}_T - \dot{q}_{NT}\right] + \left[\dot{q}_T - \dot{q}_{NT}\right]^{b+1}$$

When using the last of the expressions, aggregate growth is split between:

- *A volume component,* originating in the tradable sector productivity growth $\dot{q}_T$,
- *A redistribution component,* which equals the productivity growth differential $[\dot{q}_T - \dot{q}_{NT}]$. As in the case of fixed sector sizes, the redistribution component shows the cost of having differences in productivity growth between sectors when sector sizes do not change. As productivity growth in the tradable sector is used to subsidize labor income in the non-tradable sector, the term enters with a minus sign, indicating its drag on economic growth.
- *A restructuring compon*ent, represented by the productivity growth differential *scaled* by the sector size elasticity factor, *b+1.* The restructuring component measures how strongly the economy is able to take advantage of the incentives for structural shifts.

The stability-condition can now be expressed as

Assumption 3: $\dot{q}_T > (1-\tau_0)i - (\dot{q}_T - \dot{q}_{NT})^{b+1} + (\dot{q}_T - \dot{q}_{NT}) \equiv Q^{TR(b)}$

In order to be able to differentiate among transition economies we introduce the following definition:

DEFINITION 4: (i) *Structurally flexible :* *b>1,*

(ii) *Structurally inflexible:* *b<1.*

In what follows is the existing industrial structure important. The industrial structure is measured by the ratio between the size of the non-tradable and the size of the tradable sector of production, $\frac{1-\alpha}{\alpha}$. As the size of the tradable (non-tradable) sector increases (decreases) in the productivity growth differential, the ratio of non- tradables to tradables is, by implication, a decreasing function of the productivity growth differential. Further, how it evolves over time traces out a country's transition trajectory.

In the model a transition economy is characterized by one sector subsidizing the other along the reasoning of the Scandinavian Model of Inflation. A particular industrial structure is described by the ratio between the subsidized and the subsidizing sector of production. This ratio is in the following referred to as the *subsidizing ratio.* The sector size elasticity on the other hand measures the change in the relative size of the subsidizing (and the corresponding change in the opposite direction of the subsidized) sector of production in response to changes in the productivity growth differential.

It should be noted that our definition of 'transition economy', i.e. $\dot{q}_T > \dot{q}_{NT}$ (Definition 3), allows for movements along the transition trajectory both in the direction of relative increase in the non-tradable sector and/or the tradable sector. The direction depends on whether the difference between sectoral productivity growth $(\dot{q}_T - \dot{q}_{NT})$ becomes bigger or smaller over the relevant time period – while remaining positive throughout in compliance with our definition of transition economy. Here we follow Blanchard (1997), in defining transition as a relative expansion of the non-tradable sector. A relative expansion of the tradable sector is then referred to as reverse transition.

Assuming a transition trajectory according to the distribution of value added between sectors, we introduce the following 'relative positions' along the transition trajectory:

DEFINITION 5: *An advanced transition economy*

$$\frac{1-\alpha}{\alpha} > 1$$

An economy where the size of the non-tradable sector outweighs the size of the tradable sector is referred to as an advanced transition economy.

DEFINITION 6: *A newcomer to transition*

$$\frac{1-\alpha}{\alpha} < 1$$

An economy where the distribution of value added is dominated by the tradable sector is referred to as a newcomer to transition, simply because it is assumed that the economy has been exposed to the forces underlying the transition process for a short period of time.

In what follows the relationship between the sector size elasticity and an economy's position on the transition trajectory - as represented by its industrial structure – does matter. Two questions are of interest: First, whether the sector size elasticity exceeds unity, and second, whether it exceeds the ratio between sector sizes defined above. This allows us to distinguish between four regimes among transition economies:

Table 1: Structure and structural flexibility in transition economies

	$\frac{1-\alpha}{\alpha} < 1$	$\frac{1-\alpha}{\alpha} > 1$
b>1	***Speedy newcomer***	***Advanced and dynamic***
b<1	***Slow starter***	***Advanced and stagnant***

Among the newcomers we distinguish between speedy newcomers and slow starters. A *speedy newcomer* is characterized by a sector size elasticity exceeding unity and the tradable sector dominating the distribution of value added. A *slow starter* is also characterized by the tradable sector dominating the distribution of value added, but carries with it a weaker response to the productivity growth differential. A transition economy characterized by sector sizes responding in the same weak manner as in a slow newcomer, but where the non-tradable sector dominates the distribution of value added, is referred to as *advanced and stagnant.* An economy where the sector size response exceeds unity and the non-tradable sector dominates the distribution of value added is referred to as *advanced and dynamic.*

For a *speedy newcomer* we know that $b > \frac{1-\alpha}{\alpha}$. Likewise, for an *advanced and stagnant* economy we know that $b < \frac{1-\alpha}{\alpha}$. For an *advanced and dynamic economy* both the ratio of non-tradables to tradables and the sector size elasticity is larger than one, but the relation between the two is uncertain. Likewise, for a *slow starter* we know that both the ratio and the sector size elasticity are less than one, but the relation between the two is once again uncertain and basically an empirical question.

In the following we will make the simplifying assumption that, $b > \frac{1-\alpha}{\alpha}$, for an *advanced and dynamic* transition economy, and that $b < \frac{1-\alpha}{\alpha}$ for a *slow starter*. Differentiating among transition economies with equal industrial structures according to structural flexibility allows us to differentiate among transition economies according to both these characteristics: structural flexibility and industrial structure. Our simplifying assumption allows the effect of structural flexibility to dominate the effect of industrial structure in the reasoning that follows.

In both a speedy newcomer and in an advanced and dynamic transition economy the increase in the subsidizing sector, α, accompanying an increase in the productivity growth differential exceeds the existing subsidizing ratio, reducing the value of the latter, and thereby reducing the subsidizing burden of each unit of production in the subsidizing sector. This allows for a larger proportion of the productivity growth differential to be used for improving the allocation of resources. Economies characterized by this relation are able to change their structural composition faster and keep pace in the transition process. When the relation between the change in the relative size of the subsiding sector and the subsidizing ratio is the opposite the transition process is difficult to get going (a slow starter), or eventually loses momentum (an advanced and stagnant transition economy). As the industrial structure varies along the transition trajectory, so does the critical value of the sector size elasticity necessary to keep the transition process on track.

When considering the impact of the sector size elasticity and a transition economy's ability to take advantage of a given set of incentives, the following statement can be argued:

Lemma 6: *Assuming equal incentives for structural shifts among transition economies it is harder for a structurally flexible economy ($b>1$) than for a structurally inflexible economy ($b<1$) to ensure stability.*

Proof: In the Appendix .

Intuitively this result owes to the fact that restructuring uses resources and does not come without opportunity cost in the short term. The benefits from restructuring are not obtained immediately but accrue only over the longer term. It is the long term that we now turn to. When sector sizes are allowed to change the debt-to-GDP ratio in a transition economy can be expressed as

$$(10) \qquad \lim_{t\to\infty}\left(\frac{D_t}{Y_t}\right) = \frac{a}{\dot{q}_T + (\dot{q}_T - \dot{q}_{NT})^{b+1} - \left(\dot{q}_T - \dot{q}_{NT}\right) - (1-\tau_0)i}.$$

Equation (10) shows how the *restructuring component* reduces the long run debt to-GDP ratio while the *redistribution component* increases the long run debt-to-GDP ratio. Hence, the sustainable debt-to-GDP ratio may differ between transition economies and depends on the relation between redistribution and restructuring.

Distinguishing between the *incentives for structural shifts* and the economy's *ability to take advantage of these incentives* provides a more differentiated understanding of factors making transition economies more able to, or more in need of, a higher debt-to-GDP ratio.

Proposition 2: *The sustainable debt-to-GDP ratio increases with the degree of structural flexibility. How it responds to incentives for structural shifts depends on both the prevailing industrial structure and the structural flexibility of the economy.*

i.e. (i) $$\frac{d\left(\lim_{t\to\infty}\dfrac{D_t}{Y_t}\right)}{db} = \left(\frac{-a(\dot{q}_T - \dot{q}_{NT})^{b+1}\ln\left(\dot{q}_T - q_{NT}\right)}{\left(\dot{q}_T + (\dot{q}_T - \dot{q}_{NT})^{b+1} - (\dot{q}_T - \dot{q}_{NT}) - (1-\tau_0)i\right)^2}\right) > 0.$$

(ii) $$\frac{d\left(\lim_{t\to\infty}\dfrac{D_t}{Y_t}\right)}{d\left((\dot{q}_T - \dot{q}_{NT})\right)} = \left(\frac{-a[(b+1)(\dot{q}_T - \dot{q}_{NT})^{b} - 1]}{\left(\dot{q}_T + (\dot{q}_T - \dot{q}_{NT})^{b+1} - (\dot{q}_T - \dot{q}_{NT}) - (1-\tau_0)i\right)^2}\right)\begin{cases} < 0 \text{ if } b > \left(\dfrac{1-\alpha}{\alpha}\right) \\ > 0 \text{ if } b < \left(\dfrac{1-\alpha}{\alpha}\right) \\ = 0 \text{ if } b = \left(\dfrac{1-\alpha}{\alpha}\right) \end{cases}$$

Proof: See appendix.

First of all, for a given set of incentives for structural shifts, the long run debt-to-GDP ratio increases unambiguously in the sector size elasticity. The more flexible the economy, the better suited it is to take advantage of the incentives for structural shifts. Flexibility stimulates the growth potential of the economy and increases the long run debt-to GDP ratio the economy can carry. How the long run debt-to-GDP ratio responds to a change in the incentives for restructuring, given a fixed flexibility level *b,* depends on the flexibility level relative to the prevailing industrial structure. This implies that incentives for restructuring do different things to the sustainable debt-to-GDP ratio at different stages on the transition trajectory depending on the level of flexibility. Stronger incentives may raise the debt-to-GDP ratio or they may reduce it. A third alternative is that the incentives have no effect at all. This renders assessments of debt service capacity a relatively complex matter, on which we elaborate in the two corollaries below.

Corollary 1 *Structural flexibility matters. That is, while structurally flexible economies require a relatively higher tradable sector productivity growth rate to ensure stability for a given productivity growth differential, increased flexibility also implies a higher sustainable debt to-GDP ratio.*

Second - when it comes to determining how the sustainable debt-to-GDP ratio is affected by increased incentives for structural shifts, both an economy's structural flexibility and its existing distribution of value added between sectors of production matter. When the sector size elasticity equals the subsidizing ratio, the debt servicing ability is unaffected by increased incentives. Otherwise, debt servicing ability is affected. Even so, as transition brings about changes in industrial structure the impact on sustainable debt-to-GDP ratios of stronger incentives for structural shifts varies according to a country's position on the transition trajectory.

Corollary2 *Industrial structure matters. In transition economies where the industrial structure differs will increased incentives for structural shifts entail different implications for debt servicing ability despite the countries being equally flexible. A higher productivity growth differential will lower the long run sustainable debt-to GDP ratio for economies where the sector size elasticity exceeds the ratio between the non-tradable and the tradable sector (the subsidizing ratio). When the sector size elasticity is smaller than the same ratio, increased incentives allow for a higher debt-to GDP ratio. While the former situation characterizes speedy newcomers as well as advanced and dynamic transition economies, the latter comes about as newcomers are slow or advanced transition economies are stagnant.*

Corollary 2 comes about as the sector size elasticity is exogenous to the model, and hence unaffected by the incentives for structural shifts an economy is exposed to. When incentives increase the economy's ability to transform is unaffected. In economies where the sector size elasticity exceeds the subsidizing ratio, the necessary debt-to GDP ratio falls, while in economies where the subsidizing ratio exceeds the economy's ability to transform, the necessary debt-to GDP ratio increases.

Integrating the impact on the long run debt-to-GDP ratio from both an increase in the incentives for transition and an increase in the ability to take advantage of these incentives gives us the following:

Theorem: *The joint impact on the sustainable debt-to-GDP ratio from incentives for restructuring, and the ability to exploit them, depends on the relation between the structure of the economy, its structural flexibility and the incentives for structural shifts it is exposed to in the following way:*

The joint impact is positive if

(i) $$b < \left(\frac{1-\alpha}{\alpha} \right) - \left(\dot{q}_T - \dot{q}_{NT} \right) \ln\left(\dot{q}_T - \dot{q}_{NT} \right)$$

and it is negative if

(ii) $$b > \left(\frac{1-\alpha}{\alpha} \right) - \left(\dot{q}_T - \dot{q}_{NT} \right) \ln\left(\dot{q}_T - \dot{q}_{NT} \right)$$

(Proof in Appendix)

The joint impact is more likely to be positive the stronger the incentives and the smaller the share of value added in the tradable sector.

Thus both a country's *position* on the transition trajectory, the *speed* at which it moves along the trajectory and the incentives for structural shifts an economy is exposed to should be taken into consideration when assessing debt sustainability.

This motivates our distinction between newcomers to transition and advanced transition economies on the one hand, focusing on countries different positions on the trajectory, and our separation of both newcomers and advanced transition economies respectively according to their structural flexibility - a proxy for how fast countries move along the trajectory.

The following assessments are allowed:

(i) When comparing advanced transition economies to newcomers, the former are more likely to see the joint impact to be positive.

(ii) When considering the incentives for transition, the joint impact is more likely to be positive the stronger the incentives.

(iii) When considering the ability to exploit incentives, the exogenous nature of the sector size elasticity makes the joint impact more likely to be positive the smaller the elasticity.

The model also carries with it a "tipping point" where the joint impact changes, and complicates the assessments regarding debt servicing ability even further. For a given level of structural flexibility and a fixed increase in the productivity growth differential over the transition trajectory, the impact on debt servicing ability might switch from negative to positive as the size of the tradable sector (the non-tradable sector) is reduced (increased). The sustainable debt-to GDP ratios might therefore first of all differ between transition economies.

Second, as individual countries move along the transition trajectory the sustainable debt-to-GDP ratio might change.

Comparing between transition- and mature economies

When returning to comparing transition economies to mature economies, but this time incorporating flexible sector sizes, the results of section 3.B are expanded and the origins of the differences are becoming more transparent.

Comparing this stability condition for transition economies with the condition for stability in a mature economy allows us the following statement:

Lemma 7: *Under conditions of equality of interest rates and tax rates the stability threshold for tradable sector productivity growth that ensures stability in a transition economy is higher than that in a mature economy,* $Q^{TR(b)} > Q^{MAT}$.

Proof: This follows from Definitions 1 and 3 and the assumptions on $\dot{q}_T$, $\dot{q}_{NT}$. (See appendix for a formal proof)

Lemma 7 is the flexible sector size analog to Lemma 1, and shows that also when sector sizes are flexible, a transition economy requires a higher productivity growth rate in its tradable sector to maintain stability, in comparison to a mature economy.

The difference between the stability threshold rates in a mature and an in a transition economy depends on both the incentives for structural shifts, $(\dot{q}_T - \dot{q}_{NT})$, existing in transition economies, and the ability, *b,* to take advantage of these incentives.

The partial impact of these two factors on the gap between the stability thresholds for a mature and a transition economy can be reasoned as follows: For a given value of structural flexibility, *b*, the gap decreases in the productivity growth differential when the structural change accompanying the productivity growth differential is dominated by the subsidizing ratio. Stated differently, if the increase in the subsidizing sector of production, α, is smaller than the existing subsidizing ratio, the gap is reduced. As the increase in the subsidizing sector is less than the subsidizing ratio, the subsidizing burden is increased. A larger part of the productivity growth differential now has to be spent on subsidizing labor than before, reducing the intensity of transition and hence also the tradable sector productivity growth rate necessary for ensuring stability. Conversely, the gap grows for transition economies where the increase in the subsidizing sector exceeds the subsidizing ratio, reducing the subsidizing burden and increasing the intensity of transition.

For determining whether the 'productivity threshold gap' between transition- and mature economies increases or decreases the relation between sector size elasticity, *b,* and subsidizing ratio $\frac{1-\alpha}{\alpha}$ is crucial. This is formalized in the following.

Lemma 8:

Given a fixed but arbitrary sector size elasticity, b, the effect of a transition economy's sectoral difference in productivity growth $\left(\dot{q}_T - \dot{q}_{NT}\right)$ *on the gap between its stability threshold and that of a mature economy (*$\mathrm{Q}^{\mathrm{TR(b)}} - \mathrm{Q}^{\mathrm{MAT}}$*) is as follows: The gap decreases when the subsidizing ratio is smaller than the sector size elasticity. The gap increases when the subsidizing ratio exceeds the sector size elasticity.*

Proof: See appendix.

Lemma 8 tells that the 'productivity threshold gap' decreases for transition economies with relatively smaller subsidizing ratios. In this case relatively less is spent on subsidizing the less efficient sector of production and relatively more is spent on restructuring the economy – so restructuring dominates subsidizing (redistribution). Conversely the gap increases for transition economies with relatively larger subsidizing ratios. In such a case relatively more is spent on subsidies and relatively less is spent on restructuring – so subsidizing (redistribution) dominates restructuring and the economy is in need of higher productivity growth in the tradable sector to finance the subsidies while maintaining stability.

Taking the sector size elasticity- and thereby the economy's ability to take advantage of a given set of incentives for structural shifts - as fixed, Lemma 8 states that for both speedy newcomers as well as for advanced and dynamic transition economies the following reasoning applies: the stronger the incentives, the more intense is transition, and the higher is the tradable sector productivity growth rate necessary for stability.

Conversely, for both slow starters and for advanced and stagnant transition economies the argument is reversed: the stronger the incentives, the less intense is transition, and the lower is the productivity growth rate threshold necessary for ensuring stability.

In the following we examine how the relative magnitudes of restructuring versus redistribution impact on the debt-to-GDP ratios.

Lemma 9: *Under conditions of equality of interest rates, tax rates and borrowing rates the sustainable debt-to-GDP ratio is equal in a mature and in a transition economy if the difference between their respective tradable sector productivity growth rates equals the difference between redistribution and restructuring in the transition economy, i.e. if*

$$\dot{q}_T^{\mathrm{TR(b)}} - \dot{q}_T^{\mathrm{MAT}} = \left[\left(\dot{q}_T - \dot{q}_{NT} \right) - \left(\dot{q}_T - \dot{q}_{NT} \right)^{b+1} \right] \equiv \mathrm{K}$$

In the following *K is interpreted as the 'restructuring threshold'*.

It follows from (6) and (10) that if $q_T^{\mathrm{TR(b)}} - q_T^{\mathrm{MAT}} = \left[(\dot{q}_T - \dot{q}_{NT}) - (\dot{q}_T - \dot{q}_{NT})^{b+1}\right] \equiv \mathrm{K}$

$$\lim_{t\to\infty}\left(\frac{D_t}{Y_t}\right)\Bigg|_{Q^{MAT}=Q^{TR(b)}-K}^{MAT} = \lim_{t\to\infty}\left(\frac{D_t}{Y_t}\right)^{TR}$$

A formal proof is in the Appendix.

The condition of equality in Lemma 9 provides a 'tipping point' in how the debt-to-GDP ratios of mature and transition economies relate to each other, as shown in the following:

Lemma 10: *Under conditions of equality of interest rates, tax rates and borrowing rates: If the difference between the respective tradable sector productivity growth rates of mature and transition economy exceeds the restructuring threshold the debt-to-GDP ratio in a transition economy is smaller than that of a mature economy. If the respective difference is smaller than the threshold K, the debt-to-GDP ratio of a transition economy is higher than that of a mature economy*

It follows again from (6) and (10) that if $q_T^{\mathrm{TR(b)}} - q_T^{\mathrm{MAT}} > K$

$$\lim_{\mathrm{t}\to\infty}\left(\frac{\mathrm{D_t}}{\mathrm{Y_t}}\right)\Bigg|_{\mathrm{Q^{MAT}<Q^{TR(b)}-K}}^{\mathrm{MAT}} > \lim_{\mathrm{t}\to\infty}\left(\frac{\mathrm{D_t}}{\mathrm{Y_t}}\right)^{\mathrm{TR}}$$

Likewise, if $q_T^{\mathrm{TR(b)}} - q_T^{\mathrm{MAT}} < K$

$$\lim_{\mathrm{t}\to\infty}\left(\frac{\mathrm{D_t}}{\mathrm{Y_t}}\right)\Bigg|_{\mathrm{Q^{MAT}>Q^{TR(b)}-K}}^{\mathrm{MAT}} < \lim_{\mathrm{t}\to\infty}\left(\frac{\mathrm{D_t}}{\mathrm{Y_t}}\right)^{\mathrm{TR}}$$

Proof: See appendix for a formal proof.

The first case would typically apply to structurally inflexible economies, while we expect to see the second in economies characterized by structural flexibility.

Leaving aside the margin based assessments and assuming that stability is fulfilled for both a mature and a transition economy the model highlights the general implications of productivity growth differentials, structural flexibility and the ability to exploit the incentives for a country's debt servicing ability, along the lines given by Proposition 1, but this time incorporating structural shifts.

Proposition 3: *Suppose a common interest rate, tax rate and borrowing rate, as well as a common tradable sector productivity growth rate,* $\bar{q}_T$ *, in mature and transition economies, which satisfies the stability condition of Assumption 3.*

$$\text{Then } \lim_{t\to\infty}\left(\frac{D_t}{Y_t}\right)^{TR} > \lim_{t\to\infty}\left(\frac{D_t}{Y_t}\right)^{MAT}$$

Proof: Se appendix

Proposition three argues that as long as there are incentives for structural shifts in place and economies are able to take advantage of these incentives, the debt servicing ability of transition economies exceeds that of mature economies lacking the same incentives. The increased debt servicing ability of transition economies is not conditional on tradable sector productivity growth catching up. Basically, it is a result of productivity growth differences between sectors changing the composition of GDP. The gradually increasing efficiency in the use of resources accompanying this unbalanced growth increases the debt servicing ability of transition economies.

4. Summary and Discussion

This paper's key finding is that transition economies distinguish themselves in matters of debt sustainability in surprising ways. They can sustain higher debt relative to GDP compared to mature market economies – especially so if their structural flexibility is high. Moreover, not only *can* they sustain higher debt-to-GDP ratios but in some cases they *should* do so as well.

Our theoretical point of departure is the Domar (1944) approach to dynamic stability assessment of the public debt/GDP ratio. We extend the Domar model by introducing two sectors of production – a sector that produces tradables and one that produces non-tradables – along the lines of the Balassa-Samuelson effect. The economics of the story is along the lines of the Scandinavian Model of Inflation where one sector is subsidizing the other.

We introduce a further extension by endogenizing relative sector sizes to allow for structural shifts. We define a mature market economy to be characterized by balanced growth across the two sectors. By contrast, we define a transition economy to be characterized by unbalanced growth between sectors.

Combining the two-sector Domar model with productivity growth differences between sectors allows us to integrate the Balassa-Samuelson effect with the condition for fiscal stability. This, in turn, allows us to relate structural inflation, public debt, transition and growth in a way that, to best of our knowledge, is novel.

It is the unequal growth in transition economies relative to the balanced growth in mature economies that is the core driver of differences in debt sustainability. At issue are different channels of growth. While the 'volume effects' are similar between mature and transition economies, transition economies receive an additional 'energy boost' from the potential shifts in the distribution of value added between sectors. Transition economies are exposed to incentives for structural shifts, defined in terms of differences in productivity growth between sectors. Advantage may or may not be taken of these incentives.If the economy is able to take advantage of the incentives and change its economic structure, one can argue that the economy *should* be allowed a higher debt-to-GDP ratio due to the increase in expected growth that accompanies the changing composition of growth. If, on the other hand, the economy is not able to take advantage of the incentives for structural shifts and spends the value added on subsidies, the economy *could* (still) but *should not* carry a higher debt-to GDP ratio, as there are no changes in expected growth.

The question is *why* a transition economy- irrespective of whether it should or not- can carry a higher debt-to-GDP burden than a mature economy? The model brings into consideration several features characterizing transition economies that are neglected in the literature.

First of all, growth is unbalanced. This offers incentives for efficiency enhancing re-allocation of factors of production as well as shifts toward a more ‘fitting’ industrial structure – the core processes of transition.

These incentives by themselves, while necessary, are not sufficient for structural shifts to actually take place. What is needed in addition is the economy’s readiness to exploit the incentives. This ‘readiness’ is in the model operationalized by the concept of structural flexibility. It is a measure of the degree of structural change in response to incentives provided by unbalanced growth.

While a structurally ***in***flexible transition economy is shown to sustain a higher debt-to-GDP ratio compared to a mature economy due to exploiting the incentives for *reallocation* of its resources, a structurally flexible economy attains additional support for a higher debt-to-GDP ratio via exploiting incentives for industrial *restructuring*.

Thus transition economies can be stratified according to their degree of structural flexibility, with the more flexible ones supporting a higher debt-to-GDP ratio relative to the less flexible ones.

Still, this is not the whole story. The second factor that enters the scene is the *prevailing industrial structure*. Given a particular level of structural flexibility, how an economy’s sustainable debt-to-GDP ratio responds to the incentives for restructuring depends also on the prevailing industrial structure. More precisely, it depends on the relation between a country’s structural flexibility and its structural status quo. This implies that incentives for restructuring do different things to the sustainable debt-to-GDP ratio depending on both an economy’s structural flexibility and its prevailing industrial structure. The combined effect on the debt-to-GP ratio can be increasing, decreasing, or have no effect at all, depending on a country’s position on the transition trajectory. So, the incentives for structural change and an economy’s ability to exploit them have quantitatively as well as qualitatively different consequences, depending on a country’s position along the transition trajectory.

This means that two transition economies that are identical with regard to incentives for restructuring as well as their ability to exploit these incentives still will differ in their sustainable debt-to-GDP ratio if they find themselves at different points of the transition path.

The combined impact on the debt-to-GDP ratio is more likely to be positive (i.e. increasing the ratio) for relatively flexible economies, as well as for economies well advanced[10] along the transition path, while it tends to be negative (reducing the ratio) for relatively *in*flexible economies and for economies at an early stage of transition.

Taking structural flexibility as a proxy for how fast an economy moves along the transition path allows the following statement: A country's position on the transition trajectory, the speed at which it moves along the trajectory and the strength of its incentives for structural change should be taken into consideration when assessing debt sustainability.

Thus the debt-to-GDP ratio will not only differ between countries, but will also vary according to one country's own progression along the transition trajectory.

This paper's model – relying on endogenous relative sector sizes – also carries with it a 'tipping point', where the joint impact of the above mentioned factors on the debt-to-GDP ratio switches from positive to negative and vice versa, giving additional complexity to the assessment of debt service ability.

Our modeling contribution to the literature consists of endogenizing relative sector sizes – thereby allowing explicitly for structural shifts - and providing a metric for the concepts of *structural flexibility* and *industrial structure*, all of which play an important role in delivering the above findings. All things considered, how much debt relative to GDP a country can (or should) sustain is highly context specific and depends on the prevailing economic structure, composition of growth, structural flexibility, and the prevailing incentives for restructuring. Hence this paper delivers a cautionary note toward the one-size-fits-all approach that, via a fixed debt-to-GDP ratio requirement, is hard-wired into the European Union's Maastricht criteria. It may, in fact, be a retarding element for transition economies. In particular, it may 'punish' exactly the better performing among them, who not only *can* sustain higher debt-to-GDP ratios, but actually *should* do so – to maintain the economy's momentum toward catching up to mature economies. More than that the model of this paper supports calls for a more differentiated set of debt stability conditions for the new EU member states aspiring to join the euro zone, taking the uniqueness of the transition process into account.

[10] Here we follow Blanchard (1997) in describing the transition process as in terms of a change from a tradable sector dominated economy toward one in which the non-tradable sector plays a bigger role.

References

Becker, S. (2011). „Public Debt in 2020: Structure Matters“, *Latin America, Current Issues,* Deutsche Bank Research, January 21, 2011.

Becker, S., Deuber G., Stankiewicz, S. (2010). "Public Debt in 2020: A sustainability analysis for DM and EM Economies.", *International Topics, Current Issues.* Deutsche Bank Research. March 24, 2010. www.dbresearch.com.

Blanchard, O. (1997). The Economics of Post-Communist Transition. Clarendon Lectures in Economics. Oxford University Press.

Blanchard, O., Chouraqui, J.C., Hagemann, R.P, and Sartor, N. (1990) "The sustainability of fiscal policy: new answers to an old question", *OECD Economic Studies*, no. 15.

Borgersen, T.-A. and King, R.M. (2009) "Balassa-Samuelson: Still haven't found what we're looking for?", Østfold University College Working Paper (Arbeidsrapport) 2009:6.

Buiter, W. (2005). "The sense and Nonsense of Maastricht revisited: What have we learned about stabilization in EMU", *CEPR Discussion Paper* DP5405 December 2005.

Corsetti, G. and Roubini, N. (1991) "Fiscal deficits, public debt, and government solvency: evidence from OECD countries", *Journal of Japanese and International Economics,* vol. 5, pp. 354-380.

De Grauwe, Paul (2009), "The politics of the Maastricht convergence criteria" Vox 15. April 2009. http://www.voxeu.org/index.php?q=node/3454

Domar, E.D. (1944) "The 'burden of debt' and the national income", *American Economic Review,* vol. 34, pp.798-827.

Égert, B., I.D.K. Lommatzsch, and C. Rault. 2002. "The Balassa-Samuelson Effect in Central and Eastern Europe: Myth or Reality?" William Davidson Institute Working Paper no. 483/2002, University of Michigan, William Davidson Institute.

European Commission (2006): Introducing the Euro: convergence criteria (12.07.2006 update) http://europa.eu/legislation_summaries/economic_and_monetary_affairs/institutional_and_economic_framework/l25014_en.htm .

Gros, D. (2003). "A stability pact for public debt?", *Centre for European Policy Studies (CEPS) Brief* No. 30, January 2003.

Haliassos, M. and Tobin, J. (1990), "The macroeconomics of government finance", in *Handbook of Monetary Economics*, Friedman, B.M. and Hahn, F.H. (eds.), Elsevier, pp. 890-959.

Hamilton, J.D. and Flavin, M.D. (1986) “ On the limitations of government borrowing: a framework for empirical texting”, *American Economic Review*, vol.76, pp.808-819.

Heise, A. (2002) “Optimal public debts, sustainable deficits, and budgetary consolidation”, *Empirica*, vol.29, pp.319-337.

Lindbeck, A., 1979. Imported and Structural Inflation and Aggregate demand: The Scandinavian Model Reconstructed, in: Lindbeck, A. (ed.), Inflation and Employment in Open Economies. Oxford University Press.

Pasinetti, L. (1998) “The myth (or folly) of the 3% deficit/GDP Maastricht parameter”, *Cambridge Journal of Economics,* vol. 22, pp.103-116.

Masson, P. (1985) “The sustainability of fiscal deficits”, *IMF Staff Papers*, vol. 21, pp.577-605.

Persson, T. (1985) “Deficits and intergenerational welfare in open economies”, *Journal of International Economics*, vol. 19, pp.67-84.

Reinhart, C and K. Rogoff (2010) “Growth in a Time of Debt”, *American Economic Review*, Vol. 100(2), pp.573-578.

Tobin, J. (1986) “The monetary-fiscal mix: long-run implications”, *American Economic Review Papers and Proceedings*, vol. 76, pp.213-218.

Spaventa, L. (1987) “The growth of public debt – sustainability, fiscal rules and monetary rules”, *IMF Staff Papers*, vol. 34, pp.374-399.

Wilcox, D.W. (1989) “The Sustainability of Government Deficits: Implications of the Present-Value Borrowing Constraint”, *Journal of Money, Credit and Banking,* 21(3), pp.291-306.

Appendix

Proof of Lemma 3

Suppose we have a mature (M) and a transition economy (TR) as given by Definitions 1 and 2, and suppose that tax rate τ_0 , interest rate i and borrowing rate a are equal across the two countries.

(i) Suppose $\dot{P}_{STR} > \dot{q}_T^{TR} - \dot{q}_T^{MAT} > 0$, equivalently $\dot{q}_T^{MAT} > \dot{q}_T^{TR} - \dot{P}_{STR}$. Subtracting $(1-\tau_0)i$ from both sides of the last inequality and dividing by the positive borrowing rate a gives

$[\dot{q}_T^{MAT} - (1-\tau_0)i]\,/\,a \;>\; [\dot{q}_T^{TR} - \dot{P}_{STR} - (1-\tau_0)i]\,/a$ and in terms of reciprocals

$a\,/\,[\dot{q}_T^{MAT} - (1-\tau_0)i] \;<\; a\,/\,[\dot{q}_T^{TR} - \dot{P}_{STR} - (1-\tau_0)i]$. By (6) and (7) the last inequality is equivalent to $\lim_{t\to\infty}\left(\frac{D_t}{Y_t}\right)^{M} < \lim_{t\to\infty}\left(\frac{D_t}{Y_t}\right)^{TR}$.

(ii) Now suppose $0 < \dot{P}_{STR} < \dot{q}_T^{TR} - \dot{q}_T^{MAT}$, equivalently $\dot{q}_T^{MAT} < \dot{q}_T^{TR} - \dot{P}_{STR}$.

Proceeding analogously to (i) leads to $\lim_{t\to\infty}\left(\frac{D_t}{Y_t}\right)^{M} > \lim_{t\to\infty}\left(\frac{D_t}{Y_t}\right)^{TR}$, as desired.

Proof of Proposition 1

By assumption the common productivity growth rate $\bar{\dot{q}}_T$ satisfies the condition of Assumption 2. By Lemma 1 this implies that $\bar{\dot{q}}_T$ also satisfies the condition of Assumption 1. So stability conditions are satisfied. Suppose we have a mature (M) and a transition (TR) economy as given by Definitions 1 and 2, and suppose that tax rate τ_0 interest rate i and borrowing rate a are equal across the two countries. By assumption we have $\dot{P}_{STR} > 0$; equivalently $0 > -\dot{P}_{STR}$. Adding $\bar{\dot{q}}_T - (1-\tau_0)i$ to both sides of the last inequality and dividing both sides by the positively valued borrowing rate a we obtain $[\bar{\dot{q}}_T - (1-\tau_0)i]\,/a >$ $[\bar{\dot{q}}_T - \dot{P}_{STR} - (1-\tau_0)i]\,/a$. In terms of reciprocals this gives

$a\,/[\bar{\dot{q}}_T - (1-\tau_0)i] < a\,/[\bar{\dot{q}}_T - \dot{P}_{STR} - (1-\tau_0)i]$. By (6) and (7) the last inequality is equivalent to $\lim_{t\to\infty}\left(\frac{D_t}{Y_t}\right)^{M} < \lim_{t\to\infty}\left(\frac{D_t}{Y_t}\right)^{TR}$, or, equivalently $\lim_{t\to\infty}\left(\frac{D_t}{Y_t}\right)^{TR} > \lim_{t\to\infty}\left(\frac{D_t}{Y_t}\right)^{M}$ ∎

Proof of Lemma 5

This amounts to showing that the derivative of the relative size of the non-tradable sector $(1-\alpha)$ is

(i) positive with regard to sector size elasticity b, and

(ii) negative with regard to the sectoral productivity growth difference $\dot{q}_T - \dot{q}_{NT}$.

(i): Using (8b) $\frac{\partial}{\partial(b)}[1-(\dot{q}_T - \dot{q}_{NT})^b] = -(\dot{q}_T - \dot{q}_{NT})^b \ell n(\dot{q}_T - \dot{q}_{NT}) > 0$, where the inequality follows from definition 3 and the assumption that $\dot{q}_T, \dot{q}_{NT} \in (0,1)$, which together imply that $(\dot{q}_T - \dot{q}_{NT}) \in (0,1)$, and the properties of natural log.

(ii) $\frac{\partial}{\partial(\dot{q}_T - \dot{q}_{NT})}[1-(\dot{q}_T - \dot{q}_{NT})^b] = -b(\dot{q}_T - \dot{q}_{NT})^{b-1} < 1$, where the inequality follows from Definition 3 and the positivity assumption on *b*. ■

Proof of Lemma 6

The proof consists of taking the derivative with respect to *b* of the right hand side of assumption 3 (the stability threshold for a transition economy with flexible sector sizes) and noting that it is positively valued.

$\frac{d\left((1-\tau_0)i - (\dot{q}_T - \dot{q}_{NT})^{b+1} + (\dot{q}_T - \dot{q}_{NT})\right)}{db} = (-)\left[(\dot{q}_T - \dot{q}_{NT})^{b+1} \ln(\dot{q}_T - \dot{q}_{NT})\right] > 0$, where the last inequality owes to the fact that, by assumption, $\dot{q}_T$, $\dot{q}_{NT} \in (0,1)$ with $\dot{q}_T > \dot{q}_{NT}$, which, in turn, implies that $(\dot{q}_T - \dot{q}_{NT}) \in (0,1)$ and consequently $\ln(\dot{q}_T - \dot{q}_{NT}) < 0$. ■

Alternative proof of Lemma 6 (without taking derivative).

By assumption, $\dot{q}_T$, $\dot{q}_{NT} \in (0,1)$ with $\dot{q}_T > \dot{q}_{NT}$, which, in turn, implies that $(\dot{q}_T - \dot{q}_{NT}) \in (0,1)$.

Consider two distinct values for sector size elasticity, b>0 and b´<0 respectively. Since $(\dot{q}_T - \dot{q}_{NT}) \in (0,1)$ it follows that $(\dot{q}_T - \dot{q}_{NT})^b < (\dot{q}_T - \dot{q}_{NT})^{b'}$. Multiplying both sides of the inequality by the positively valued quantity $(\dot{q}_T - \dot{q}_{NT})$ we obtain $(\dot{q}_T - \dot{q}_{NT})^{b+1} < (\dot{q}_T - \dot{q}_{NT})^{b'+1}$, which, upon multiplying both sides of the inequality by (-1) becomes

$-(\dot{q}_T - \dot{q}_{NT})^{b+1} > -(\dot{q}_T - \dot{q}_{NT})^{b'+1}$. Adding $(\dot{q}_T - \dot{q}_{NT})$ and $(1-\tau_0)i$ to both sides of the inequality gives $(1-\tau_0)i - (\dot{q}_T - \dot{q}_{NT})^{b+1} + (\dot{q}_T - \dot{q}_{NT}) >$ $(1-\tau_0)i - (\dot{q}_T - \dot{q}_{NT})^{b'+1} + (\dot{q}_T - \dot{q}_{NT})$.

By assumption 3 this is equivalent to saying that the stability threshold for the case b>1 is higher than that with b<1.

Proof of Lemma 7

By assumption we have (i) $\dot{q}_T$, $\dot{q}_{NT} \in (0,1)$ and (ii) $\dot{q}_T > \dot{q}_{NT}$ for a transition economy, which together imply

(iii) $(\dot{q}_T - \dot{q}_{NT}) \in (0,1)$. Since by definition sector size elasticity b is positive, (iii) implies

$(\dot{q}_T - \dot{q}_{NT}) > (\dot{q}_T - \dot{q}_{NT})^{b+1}$. Subtracting $(\dot{q}_T - \dot{q}_{NT})^{b+1}$ from both sides of the inequality and adding $(1-\tau_0)i$ we obtain $(1-\tau_0)i - (\dot{q}_T - \dot{q}_{NT})^{b+1} + (\dot{q}_T - \dot{q}_{NT}) > (1-\tau_0)i.$

By assumption 1 and 3 this is equivalent to the statement $Q^{TR(b)} > Q^{MAT}$ as claimed. ■

Proof of Lemma 8

This amounts to taking the derivative of $(Q^{TR(b)} - Q^{MAT})$ with respect to $(\dot{q}_T - \dot{q}_{NT})$ and finding the conditions for it to be negatively and positively valued.

However, since Q^{MAT} does not depend on $(\dot{q}_T - \dot{q}_{NT})$ it suffices to take the derivative of $Q^{TR(b)}$ as given in assumption 3, with respect to $(\dot{q}_T - \dot{q}_{NT})$ and finding the conditions for it to be

(i) negatively and (ii) positively valued.

(i) $\frac{\partial}{\partial(\dot{q}_T - \dot{q}_{NT})}\left((1-\tau_0)i - (\dot{q}_T - \dot{q}_{NT})^{b+1} + (\dot{q}_T - \dot{q}_{NT})\right) < 0$ implies $-(b+1)\ (\dot{q}_T - \dot{q}_{NT})^{b} + 1 < 0.$

By the definition of α and re-arranging this is equivalent to $-(b+1)\,\alpha < -1$, which upon multiplying out the left hand side of the inequality and multiplying both sides by (– 1) gives

$b\,\alpha + \alpha > 1$; equivalently $b > \left(\frac{1-\alpha}{\alpha}\right)$.

In an analogous manner it can be shown that

(ii) $\frac{\partial}{\partial(\dot{q}_T - \dot{q}_{NT})}\left((1-\tau_0)i - (\dot{q}_T - \dot{q}_{NT})^{b+1} + (\dot{q}_T - \dot{q}_{NT})\right) > 0$ implies $b < \left(\frac{1-\alpha}{\alpha}\right)$.

The statement about decreasing and/or widening gap relative to a mature economy follows directly from Lemma 7. ■

Proof of Proposition 2

This amounts to taking two partial derivatives of the sustainable debt-to-GDP ratio:

(i) with respect to the sector size elasticity, *b,* and (ii) with respect to the difference in sectoral productivity growth $(\dot{q}_T - \dot{q}_{NT})$.

(i)

$$\frac{d\left(\lim_{t\to\infty}\frac{D_t}{Y_t}\right)}{db} = \frac{d}{db}\left(\frac{a}{\left(\dot{q}_T + (\dot{q}_T - \dot{q}_{NT})^{b+1} - (\dot{q}_T - \dot{q}_{NT}) - (1-\tau_0)i\right)}\right)$$

$$= \left(\frac{-a(\dot{q}_T - \dot{q}_{NT})^{b+1}\ln(\dot{q}_T - q_{NT})}{\left(\dot{q}_T + (\dot{q}_T - \dot{q}_{NT})^{b+1} - (\dot{q}_T - \dot{q}_{NT}) - (1-\tau_0)i\right)^2}\right) > 0,$$

where the last inequality owes to the assumptions that

$\dot{q}_T$, $\dot{q}_{NT} \in (0,1)$ and $\dot{q}_T > \dot{q}_{NT}$, which, in turn, implies that $(\dot{q}_T - \dot{q}_{NT}) \in (0,1)$, and consequently

$\ln(\dot{q}_T - q_{NT}) < 0$.

(ii)

$$\frac{d\left(\lim_{t\to\infty}\frac{D_t}{Y_t}\right)}{d((\dot{q}_T-\dot{q}_{NT}))}=\frac{d}{d(\dot{q}_T-\dot{q}_{NT})}\left(\frac{a}{\left(\dot{q}_T+(\dot{q}_T-\dot{q}_{NT})^{b+1}-(\dot{q}_T-\dot{q}_{NT})-(1-\tau_0)i\right)}\right)$$

$$=\left(\frac{-a[(b+1)(\dot{q}_T-\dot{q}_{NT})^b-1]}{\left(\dot{q}_T+(\dot{q}_T-\dot{q}_{NT})^{b+1}-(\dot{q}_T-\dot{q}_{NT})-(1-\tau_0)i\right)^2}\right)$$

For the sign of this expression the sign of the numerator $-a[(b+1)(\dot{q}_T-\dot{q}_{NT})^b-1]$ is decisive.

$-a[(b+1)(\dot{q}_T-\dot{q}_{NT})^b-1]<0$ if $(b+1)(\dot{q}_T-\dot{q}_{NT})^b>1$. By the definition of α the last inequality is expressed as (b+1) α > 1, or, equivalently $b>\left(\frac{1-\alpha}{\alpha}\right)$.

Analogously $-a[(b+1)(\dot{q}_T-\dot{q}_{NT})^b-1]>0$ if $(b+1)(\dot{q}_T-\dot{q}_{NT})^b<1$, or equivalently if

$b<\left(\frac{1-\alpha}{\alpha}\right)$.

Lastly $-a[(b+1)(\dot{q}_T-\dot{q}_{NT})^b-1]=0$ if $(b+1)(\dot{q}_T-\dot{q}_{NT})^b=1$, equivalently if

$b=\left(\frac{1-\alpha}{\alpha}\right)$. ■

Proof of Theorem

At issue are the two partial derivatives $\frac{\partial\left(\lim_{t\to\infty}\frac{D_t}{Y_t}\right)}{\partial b}$ and $\frac{\partial\left(\lim_{t\to\infty}\frac{D_t}{Y_t}\right)}{\partial(\dot{q}_T-\dot{q}_{NT})}$.

By proposition 2 $\frac{\partial\left(\lim_{t\to\infty}\frac{D_t}{Y_t}\right)}{\partial b}>0$, while the sign of $\frac{\partial\left(\lim_{t\to\infty}\frac{D_t}{Y_t}\right)}{\partial(\dot{q}_T-\dot{q}_{NT})}$ indeterminate and is subject to further conditions. This implies that whether or not the joint impact on the debt-to-

GDP ratio is positive or negative depends on the sign and relative magnitude of

$$\frac{\partial\left(\lim_{t\to\infty}\frac{D_t}{Y_t}\right)}{\partial(\dot{q}_T-\dot{q}_{NT})}.$$

A: If the sign is positive or zero then the joint impact on the debt-to-GDP ratio is unambiguously positive.

B: If the sign is negative then the joint impact depends on the relative magnitudes of the two partial derivatives.

a) If $\frac{\partial\left(\lim_{t\to\infty}\frac{D_t}{Y_t}\right)}{\partial b}$ dominates then the joint impact is positive.

b) If $\frac{\partial\left(\lim_{t\to\infty}\frac{D_t}{Y_t}\right)}{\partial(\dot{q}_T-\dot{q}_{NT})}$ dominates then the joint impact is negative.

It remains to find the conditions under which A, Ba, and Bb hold relevance.

A: $\frac{\partial\left(\lim_{t\to\infty}\frac{D_t}{Y_t}\right)}{\partial(\dot{q}_T-\dot{q}_{NT})}\geq 0$ implies, by Proposition2, $\frac{-a[(b+1)(\dot{q}_T-\dot{q}_{NT})^b-1]}{\left(\dot{q}_T+(\dot{q}_T-\dot{q}_{NT})^{b+1}-(\dot{q}_T-\dot{q}_{NT})-(1-\tau_0)i\right)^2}\geq 0$.

Upon multiplying both sides by the positively valued expression of the denominator and dividing both sides of the inequality by (− a), where a>0, we obtain $(b+1)(\dot{q}_T-\dot{q}_{NT})^b-1\leq 0$.

Adding 1 to both sides and dividing by the positively valued (b+1) gives $(\dot{q}_T-\dot{q}_{NT})^b\leq\frac{1}{b+1}$.

By the definition of tradable sector size α this is equivalent to $\alpha\leq\frac{1}{b+1}$. Upon multiplying both sides by (b+1), rearranging and solving for b we obtain the condition $b\leq\frac{1-\alpha}{\alpha}$. ■

B: Analogously to A it can be shown that the condition for $\frac{\partial\left(\lim_{t\to\infty}\frac{D_t}{Y_t}\right)}{\partial(\dot{q}_T-\dot{q}_{NT})}<0$ to hold requires the condition $b>\frac{1-\alpha}{\alpha}$. It remains to show under which conditions sub-cases a and b hold.

a) To show under what conditions $\frac{\partial\left(\lim_{t\to\infty}\frac{D_t}{Y_t}\right)}{\partial b}$ dominates we find the conditions under which

$$\left\|\frac{\partial\left(\lim_{t\to\infty}\frac{D_t}{Y_t}\right)}{\partial b}\right\|>\left\|\frac{\partial\left(\lim_{t\to\infty}\frac{D_t}{Y_t}\right)}{\partial(\dot{q}_T-\dot{q}_{NT})}\right\|.$$

By the definition of absolute value and the assumptions of case B this is equivalent to

$\frac{\partial\left(\lim_{t\to\infty}\frac{D_t}{Y_t}\right)}{\partial b} > -\frac{\partial\left(\lim_{t\to\infty}\frac{D_t}{Y_t}\right)}{\partial(\dot{q}_T-\dot{q}_{NT})}$, or equivalently, by Proposition 2,

$$\left(\frac{-a(\dot{q}_T-\dot{q}_{NT})^{b+1}\ln(\dot{q}_T-q_{NT})}{\left(\dot{q}_T+(\dot{q}_T-\dot{q}_{NT})^{b+1}-(\dot{q}_T-\dot{q}_{NT})-(1-\tau_0)i\right)^2}\right)>\left(\frac{a[(b+1)(\dot{q}_T-\dot{q}_{NT})^b-1]}{\left(\dot{q}_T+(\dot{q}_T-\dot{q}_{NT})^{b+1}-(\dot{q}_T-\dot{q}_{NT})-(1-\tau_0)i\right)^2}\right)$$

Multiplying both sides of the inequality by the positive expression of the denominator and dividing by $a>0$ obtains $-(\dot{q}_T-\dot{q}_{NT})^{b+1}\ln(\dot{q}_T-\dot{q}_{NT}) > (b+1)(\dot{q}_T-\dot{q}_{NT})^b-1$, or equivalently,

$-(\dot{q}_T-\dot{q}_{NT})^{b+1}\ln(\dot{q}_T-\dot{q}_{NT})-(b+1)(\dot{q}_T-\dot{q}_{NT})^b+1>0$. Multiplying out the second term on L.H.S. gives $-(\dot{q}_T-\dot{q}_{NT})^{b+1}\ln\left(\dot{q}_T-\dot{q}_{NT}\right)-b(\dot{q}_T-\dot{q}_{NT})^b-(\dot{q}_T-\dot{q}_{NT})^b+1>0$, which by the definition of α is equivalent to $-(\dot{q}_T-\dot{q}_{NT})^{b+1}\ln(\dot{q}_T-\dot{q}_{NT})-b(\dot{q}_T-\dot{q}_{NT})^b+(1-\alpha)>0$. Subtracting (1− α) from both sides, multiplying by (− 1), factoring out $(\dot{q}_T-\dot{q}_{NT})^b$ and once more applying the definition of α on L.H.S. gives $\alpha\left[(\dot{q}_T-\dot{q}_{NT})\ln(\dot{q}_T-\dot{q}_T)+b\right]<(1-\alpha)$, and upon dividing both sides by α and re-arranging we arrive at the condition

$$\frac{1-\alpha}{\alpha} > b + (\dot{q}_T - \dot{q}_{NT})\ \ln(\dot{q}_T - \dot{q}_T) \quad \text{or, alternatively,}\ \ b < \left(\frac{1-\alpha}{\alpha}\right) - (\dot{q}_T - \dot{q}_{NT})\ln(\dot{q}_T - \dot{q}_{NT})$$

■ This proves part (i) of the Theorem.

b)To show under what conditions $\frac{\partial\left(\lim_{t\to\infty}\frac{D_t}{Y_t}\right)}{\partial(\dot{q}_T - \dot{q}_{NT})}$ dominates and thus the joint impact on the debt-to-GDP ratio is negative we derive conditions under which

$$\left\|\frac{\partial\left(\lim_{t\to\infty}\frac{D_t}{Y_t}\right)}{\partial b}\right\| < \left\|\frac{\partial\left(\lim_{t\to\infty}\frac{D_t}{Y_t}\right)}{\partial(\dot{q}_T - \dot{q}_{NT})}\right\| .$$

The proof proceeds analogously to part (a), and arrives at the condition

$$\frac{1-\alpha}{\alpha} < b + (\dot{q}_T - \dot{q}_{NT})\ \ln(\dot{q}_T - \dot{q}_T), \text{ or, alternatively,}\ \ b > \left(\frac{1-\alpha}{\alpha}\right) - (\dot{q}_T - \dot{q}_{NT})\ln(\dot{q}_T - \dot{q}_{NT})$$

which gives us condition (ii) of the theorem. ■

Proof of Lemma 9

Suppose $\dot{\mathrm{q}}_{\mathrm{T}}^{\mathrm{TR}} - \dot{\mathrm{q}}_{\mathrm{T}}^{\mathrm{MAT}} = \left[(\dot{q}_T - \dot{q}_{NT}) - (\dot{q}_T - \dot{q}_{NT})^{b+1}\right]$.

Adding $\dot{\mathrm{q}}_{\mathrm{T}}^{\mathrm{MAT}}$ and $(\dot{q}_T - \dot{q}_{NT})^{b+1}$ to both sides of the equality and subtracting $(\dot{q}_T - \dot{q}_{NT})$ and $(1-\tau_0)i$ we obtain

$\dot{\mathrm{q}}_{\mathrm{T}}^{\mathrm{TR}} + (\dot{q}_T - \dot{q}_{NT})^{b+1} - (\dot{q}_T - \dot{q}_{NT}) - (1-\tau_0)i = \dot{\mathrm{q}}_{\mathrm{T}}^{\mathrm{MAT}} - (1-\tau_0)i$, implying the reciprocal equality

$\left(\frac{1}{\dot{\mathrm{q}}_{\mathrm{T}}^{\mathrm{TR}} + (\dot{q}_T - \dot{q}_{NT})^{b+1} - (\dot{q}_T - q_{NT}) - (1-\tau_0)i}\right) = \left(\frac{1}{\dot{\mathrm{q}}_{\mathrm{T}}^{\mathrm{MAT}} - (1-\tau_0)i}\right)$, which, upon multiplying both sides by the positively valued borrowing rate a gives

$$\frac{a}{\left(\dot{\mathrm{q}}_{\mathrm{T}}^{\mathrm{TR}}+(\dot{q}_T-\dot{q}_{NT})^{b+1}-(\dot{q}_T-\dot{q}_{NT})-(1-\tau_0)i\right)}=\frac{a}{\left(\dot{\mathrm{q}}_{\mathrm{T}}^{\mathrm{MAT}}-(1-\tau_0)i\right)}$$. By (6) and (10) the last inequality is equivalent to $\lim_{t\to\infty}\left(\frac{D_t}{Y_t}\right)^{TR}=\lim_{t\to\infty}\left(\frac{D_t}{Y_t}\right)^{M}$ ■

Proof of Lemma 10

Suppose $\left[(\dot{q}_T-\dot{q}_{NT})-(\dot{q}_T-\dot{q}_{NT})^{b+1}\right]>K$. By the definition of K this is equivalent to the inequality $\dot{\mathrm{q}}_{\mathrm{T}}^{\mathrm{TR}}-\dot{\mathrm{q}}_{\mathrm{T}}^{\mathrm{MAT}}<\left[(\dot{q}_T-\dot{q}_{NT})-(\dot{q}_T-\dot{q}_{NT})^{b+1}\right]$.

Adding $\dot{\mathrm{q}}_{\mathrm{T}}^{\mathrm{MAT}}$ and $(\dot{q}_T-\dot{q}_{NT})^{b+1}$ to both sides of the inequality and subtracting $(\dot{q}_T-\dot{q}_{NT})$ and $(1-\tau_0)i$ we obtain $\dot{\mathrm{q}}_{\mathrm{T}}^{\mathrm{TR}}+(\dot{q}_T-\dot{q}_{NT})^{b+1}-(\dot{q}_T-\dot{q}_{NT})-(1-\tau_0)i<\dot{\mathrm{q}}_{\mathrm{T}}^{\mathrm{MAT}}-(1-\tau_0)i$, implying the reciprocal equality

$$\left(\frac{1}{\dot{\mathrm{q}}_{\mathrm{T}}^{\mathrm{TR}}+(\dot{q}_T-\dot{q}_{NT})^{b+1}-(\dot{q}_T-q_{NT})-(1-\tau_0)i}\right)>\left(\frac{1}{\dot{\mathrm{q}}_{\mathrm{T}}^{\mathrm{MAT}}-(1-\tau_0)i}\right)$$, which, upon multiplying both sides by the positively valued borrowing rate *a* gives

$$\frac{a}{\left(\dot{\mathrm{q}}_{\mathrm{T}}^{\mathrm{TR}}+(\dot{q}_T-\dot{q}_{NT})^{b+1}-(\dot{q}_T-\dot{q}_{NT})-(1-\tau_0)i\right)}>\frac{a}{\left(\dot{\mathrm{q}}_{\mathrm{T}}^{\mathrm{MAT}}-(1-\tau_0)i\right)}$$. By (6) and (10) the last inequality is equivalent to $\lim_{t\to\infty}\left(\frac{D_t}{Y_t}\right)^{TR}>\lim_{t\to\infty}\left(\frac{D_t}{Y_t}\right)^{M}$.

Suppose $\left[(\dot{q}_T-\dot{q}_{NT})-(\dot{q}_T-\dot{q}_{NT})^{b+1}\right]<K$. By analogous argumentation as above it can be shown that this implies $\lim_{t\to\infty}\left(\frac{D_t}{Y_t}\right)^{TR}<\lim_{t\to\infty}\left(\frac{D_t}{Y_t}\right)^{M}$.

■

Proof of Proposition 3

Again, suppose we have a mature (M) and a transition (TR) economy as given by Definitions 1 and 2, and suppose that tax rate τ_0 interest rate i and borrowing rate a are equal across the two countries. By assumption $\dot{q}_T$, $\dot{q}_{NT} \in (0,1)$ and $\dot{q}_T > \dot{q}_{NT}$, which, in turn, implies that $(\dot{q}_T - \dot{q}_{NT}) \in (0,1)$. By assumption b>0 which implies that b+1>1. It follows that

$(\dot{q}_T - \dot{q}_{NT})^{b+1} < (\dot{q}_T - \dot{q}_{NT})$, equivalently, $(\dot{q}_T - \dot{q}_{NT})^{b+1} - (\dot{q}_T - \dot{q}_{NT}) < 0$. Adding $\dot{q}_T$ and subtracting $(1-\tau_0)i$ from both sides of the last inequality obtains

$\dot{q}_T + (\dot{q}_T - \dot{q}_{NT})^{b+1} - (\dot{q}_T - \dot{q}_{NT}) - (1-\tau_0)i < \dot{q}_T - (1-\tau_0)i$ and its implied reciprocal inequality

$\left(\frac{1}{\dot{q}_T + (\dot{q}_T - \dot{q}_{NT})^{b+1} - (\dot{q}_T - q_{NT}) - (1-\tau_0)i}\right) > \left(\frac{1}{\dot{q}_T - (1-\tau_0)i}\right)$, which, upon multiplying both sides of the inequality by the a>0 gives

$$\frac{a}{\left(\dot{q}_T + (\dot{q}_T - \dot{q}_{NT})^{b+1} - (\dot{q}_T - \dot{q}_{NT}) - (1-\tau_0)i\right)} > \frac{a}{\left(\dot{q}_T - (1-\tau_0)i\right)}$$

By (6) and (10) the last inequality is equivalent to $\lim_{t\to\infty}\left(\frac{D_t}{Y_t}\right)^{TR} > \lim_{t\to\infty}\left(\frac{D_t}{Y_t}\right)^{M}$ ■