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Trond-Arne Borgersen

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Trond-Arne Borgersen
Østfold University College
Department of Economics and Social Sciences
Remmen
1757 Halden
Norway
E-mail: trond.a.borgersen@hiof.no
Phone: +4797756261

Abstract:
Housing markets reflect our housing consumption profile over the life cycle. As we age, marry and have kids, we seek larger dwellings and to a greater extent owner-occupied housing. The up-trading process has two key characteristics: First, it is equity induced. Second, it impacts both the supply and demand sides of housing markets. This is our point of departure. The paper combines a housing ladder with a house price index to show how up-trading amplifies shocks and introduces a multiplier into the housing market. The interplay between market segments results in up-trading induced price dispersion and a price response in the segments on top of the ladder that exceeds those of segments further down, even when shocks are equal across market segments. Finally, as up-trading impacts both housing supply and housing demand even balanced shocks to net demand might impact house prices. Focusing on different market segments, both direct (the size effect) and indirect (the up-trading effect) effects impact the house price index. This paper highlights policy options at a finer level when in need of stimulating or dampening house price cycles.

Keywords: Housing careers, housing market multiplier, house price dispersion.

JEL: R21, R31.

1. Introduction

For the majority of households a housing career often takes the form of first renting when young, before buying a small block house (flat) as a young adult, trading-up to a row house when becoming parents and in need of more space, before taking another step on the housing ladder to a detached house when one can afford it as semi-old, and potentially trading down again to a block house when in retirement. Impacting both the supply and demand sides of housing markets, housing career structures might have profound implications on house prices. As housing careers differ between industrial- and post-industrial societies (Beer, 2007) the accompanying price effects might have changed over time.

The existence of housing careers brings heterogeneity into the housing market and is an argument in favour of applying a multimarket structure when analysing market developments. A multimarket structure allows one to address idiosyncratic features of the housing market instead of just analysing how changes in macroeconomic variables such as unemployment and interest rates impact housing markets. More specifically, in a multimarket structure, the interplay between market segments can be addressed, and – in relation to that –the impact of equity induced up-trading for house prices. As the exposure to households entering and leaving the housing market differs between segments, shocks to different market segments might have different implications for house prices. In particular, there might be important distinctions between shocks to the segments on the first and the final step of the housing ladder exposed to either entry or exit, and the segments in between, simultaneously exposed to both.

This paper develops a model for the housing market, taking some key characteristics accompanying housing careers into account: First, that rentals do not represent alternative housing to all types of home ownership. Second, that market segments for owner occupied housing are linked through up-trading. Third, that up-trading is equity induced. And fourth, that while nth time buyers also are sellers, first time buyers are just that.

First, we sketch a housing market with three market segments for owner-occupation and specify an up-trading process. This housing career structure is combined with a house price index where each market segment is included according to its market size. While keeping supply fixed, a housing market multiplier is derived as market segments impact the house price index both through their size (a size effect) and through their position on the housing ladder (an up-trading effect). The up-trading process amplifies the effect on the house price index in response to shocks to income, rent or mortgage availability. The model provides a potential explanation for short-term deviations in a number of the indicators used to assess housing market equilibriums. Equity induced up-trading is also the reason for why market segment prices on top of the ladder respond stronger to shocks than prices in segments further down. This is because the former is exposed to indirect effects that the latter are not. Taking the multiplier and the price dispersion between segments into account, the up-trading process provides a blueprint for housing market policies at a finer scale if needed to boost or dampen a house price cycle.

Second, to take into account that nth time buyers also are sellers we endogenise supply. Now, the multimarket approach shows the importance of taking into account whether a market segment is the first, the final or simply a transit on the way to the final destination of a housing career, when assessing the implications of shocks to supply and demand for house prices. The price effect of shocks to either the first or the final step of the housing ladder differs substantially from those of shocks to the segment in between. To highlight the features of the up-trading process residing on the supply and demand sides of housing markets respectively, we distinguish between shocks to gross and net demand. As demand has a dual role the price effects derived above might be altered. However, even when incorporating the effects of up-trading on both supply and demand we see price dispersion between market segments, a housing market multiplier and the strongest price effects when shocks reside on the first step of the housing ladder. As a result even balanced shocks to net demand might now impact house prices.

While our heterogeneous housing market model differs from more conventional homogenous housing market structures, for instance Hilbers et al. (2009), it relates to Swank (2002) arguing the interdependence between different market segments. The importance of first-time entry into housing markets, and the first steps of the housing ladder, for aggregate house prices, is related to Mankiw and Weil (1989) and Ortalo'-Magne and Rady (1999, 2005). The distinction between first-time and repeat buyers, and the importance of equity effects for the latter's housing market behaviour, is along the lines of Larsen (2010).[1] Focusing on the interplay between market segments our approach is however broader, and allows the

[1] A number of papers analyse the wealth effects arising from housing markets, amongst others, Benjamin et al. (2002), Case et al. (2002), IMF (2002), Yamashita (2007) and Larsen (2010).

distribution of equity effects between market segments to impact house prices. Our equity induced up-trading creates a housing market multiplier which introduces a financial accelerator into the housing market along the lines of Bernanke et al. (1998) and house price overreactions similar to those of Lamont and Stein (1999). In fact, when the price effects of shocks to supply and demand differ, the housing market is characterised by the same kind of features as those underlying the Balanced Budget Multiplier of Haavelmo (1966): Balanced shocks to net demand impact on house prices. The up-trading process makes prices on the upper end of the ladder appreciate stronger than prices on the first step of the ladder as in Poterba (1991), Mayer (1993) and Early (1996). The multimarket model also shows the importance of the market segments in between, simultaneously exposed to both entry and exit, for house prices, in a way that, to the best of our knowledge, is novel.

This paper is structured as follows: The next part sets out a housing market model with up-trading and market segments to illustrate our housing career structure. The third part derives a housing market multiplier based on equity induced up-trading effects in demand. The housing market multiplier is derived, and conditions for an efficient policy mix are sketched. The fourth part endogenise supply, by taking into account that up-trading makes households simultaneously act as both buyers and sellers, allowing for demand in a dual role in the presence of housing careers. The last part contains concluding remarks.

2. A housing career structure with up-trading

A housing career is most easily understood as a sequence of housing circumstances an individual or a household occupies over their life (Beer and Faulkner, 2009). Normally, housing careers are related to three key drivers: demographic shifts, changes in labour markets and shifting consumption patterns (Beer, Faulkner, and Gabriel, 2006). Beer (2007) argues housing careers in post-industrial societies are also influenced by economic liberalisation; changes in attitudes towards housing, market based housing policies, changes in attitudes towards social roles and house price growth create an asset base for home owners. In many post-industrial societies a higher frequency of moving has been accompanied by increased homeownership rates, in the U.S. by a remarkable 5 percentage points between 1994 and 2004 (Doms and Krainer, 2007). Financial liberalisation and innovations in mortgage markets have been argued as important for the increase in homeownership (Chambers, Garriga and Schlagenhauf, 2005). Lowering of down payment requirements, increased flexibility in repayment schedules and reductions in the cost associated with extracting home equity are all highlighted, see for instance Geradi, Rosen and Willen (2006). While the first two might be especially important for the increase in home ownership rates among younger and low-income households, the latter might be equally important for the ability to re-establish oneself as a home owner after adverse family-related experiences (Chambers and Garriga, 2007).

In this paper we assess the impact on market segment prices and the house price index following credit constraints and equity gains from owner-occupied housing. As in Borgersen and Sommervoll (2011), we consider a simplified version of a housing market where owner-occupation can occur in three *distinct* segments: starter (s), intermediate (m), and family homes (f). These market segments are characterised by dwellings with different characteristics.[2] Each segment is directed towards three different types of households, labelled starter (s), intermediate (m) and family (f), represented by groups with preferences for differing housing characteristics, for instance separated by age. Such a housing career structure can be related to credit constraints (Ortalo-Magne and Rady, 2006). To take into account the argument of Andrew and Heurin, (2006) of distinguishing between repeat buyers and first-time entrants into owner-occupation, we include a rental market.

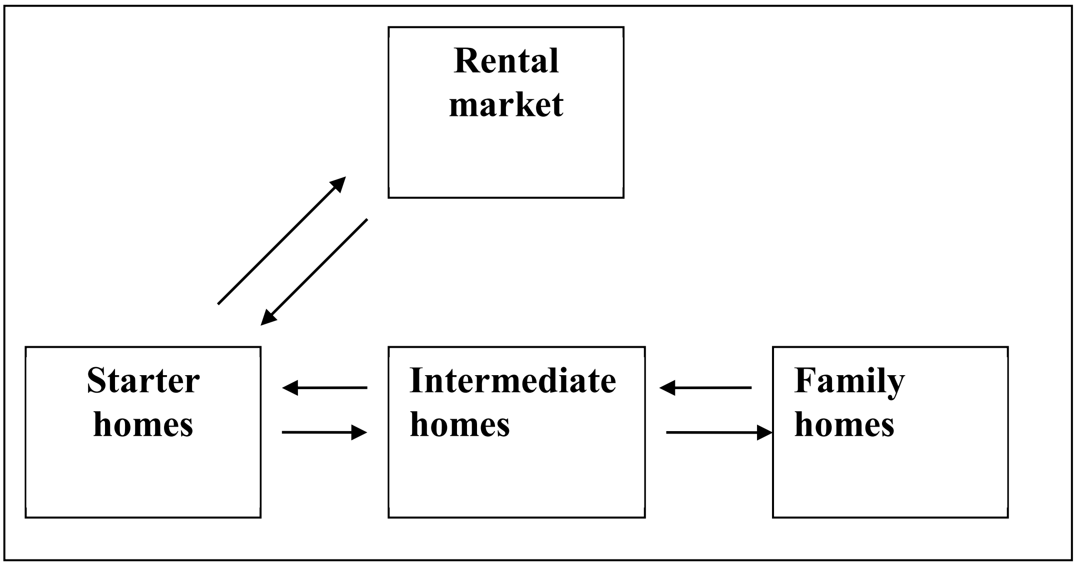
We assume that rentals are only considered an alternative to starter homes, while a housing ladder defines a specific relation between the segments for owner-occupied housing. A starter home is the first step on the ladder for owner-occupied housing. An intermediate home is the second, while a family home is the final step on the housing ladder. For intermediate homes, alternative housing is considered to be a starter home, and for family homes alternative housing is in the intermediate market. This model structure relates substitution to the market segment one step down the ladder. For credit constrained households the up-trading process is related to the equity gains from existing housing. These assumptions produce symmetry between substitution effects and equity induced up-trading. Figure 2 illustrates our presumed up-trading structure.

Up-trading links the market segments for owner-occupied housing both through the demand and supply sides of housing markets. In addition to conventional substitution effects, there is a demand side link through the equity gains accompanying housing appreciation. On the supply

[2] See Bourassa (2003) for a discussion of submarkets.

side market segments are linked by nth time buyers who are also sellers. A household demanding an intermediate (family) home simultaneously puts its existing starter (intermediate) home out for sale.

Figure 1*: The housing ladder and the up-trading process*



3. A linear housing ladder

In this section a simple stylized housing market model is derived to assess the interplay between different market segments positioned in relation to one another in a housing market ladder. To simplify we keep supply fixed.

A household's demand for owner occupied housing is in general likely to depend on the price of housing, P; household income, Y; credit supply (if credit constrained), M; and the price of alternative housing and households net equity, E, in addition to the price of alternative housing, A. In short, the demand for housing can be expressed as: D=D (P, Y, M, E, A). Without loss of generality we abstract away from substitution effects in demand.[3]

Furthermore, the heterogonous market structure is characterised by market segments linked through equity gains from home ownership. To highlight up-trading the aggregate equity entering the demand for intermediate homes is made a function of the price for starter homes: $E_m = E_m(P_s)$. Likewise, the equity component impacting family homes depends on the price of

[3] A household that is assumed to substitute *down* the ladder will carry with it a substitution effect that is symmetric to the up-trading effect highlighted in the model. See for instance Borgersen and Sommervoll (2011) for an approach where the indirect effect includes both substitution and equity induced up-trading.

intermediaries, $E_f = E_f(,P_m)$. For simplicity we consider linearized versions of the equity functions:

1) $e_m E_m = e_m E_{0m} + e_{ms} P_s$

2) $e_f E_f = e_f E_{0f} + e_{fm} P_m$

The demand for owner occupied housing is – when considering linearized demand functions as well – in each of the three submarkets:

3) $D_s = k_s + m_s M_s + e_s E_{0s} + y_s Y_s - p_s P_s$

4) $D_m = k_m + m_n M_m + e_m E_{0m} + e_{ms} P_s + y_m Y_m - p_m P_m$

5) $D_f = k_f + m_f M_f + e_f E_{0f} + e_{fm} P_m + y_f Y_f - p_f P_f$

where k_i is a constant, M_i is aggregate mortgage supply, E_i is aggregate equity, P_i is price of housing, and Y_i is household income, for i = s, m and f. In our linearization the parameters represent elasticises, and $e_i E_{0i}$ exogenously given equity components. For the nth step of the ladder, e_{ij} is an indicator of the equity induced up trading from segment j to the segment i.

Furthermore, market equilibrium for each market segment is given as:

6) $D_i = S_i$

where S_i is housing supply in market segment i. For now supply is fixed, an assumption to be relaxed in section 4.

Finally, the house price index of the housing market given by:

7) $P = \sum_i \alpha_i P_i$

Where the weights, α_i, reflects the corresponding submarkets size.

3.1 Market segment prices

By equilibrating supply and demand in each submarket we can derive expressions for the market segment prices and eventually the house price index. To simplify, price effects are analysed in terms of net demand, $ND_i = \left[c_i + m_i M_i + y_i Y_i - S_i\right]$, where $c_i = k_i + e_i E_{i0}$. Net demand is increasing in household income and mortgage availability while decreasing in supply.

Starter homes

By applying the market equilibrium condition for starter homes and inserting for demand from (3), the price on starter homes equals:

8) $$\mathrm{P_s} = \frac{1}{\mathrm{p_s}}\left[\mathrm{ND_s}\right]$$

Being the first step on the housing ladder starter home prices are only affected by factors *directly* impacting this segment. The price of starter homes is positively related to $\mathrm{ND_s}$, where supply has a conventional negative impact while household income, household equity and mortgage availability impacts positively on the price of starter homes.

Intermediate homes

Using the market segment equilibrium condition, the demand for intermediate homes (4) and the equilibrium price of starter homes in (8), the price on intermediate homes equals:

9) $$P_m = \frac{1}{p_m}\left[ND_m + \frac{e_{ms}}{p_s}\left[ND_s\right]\right]$$

While starter home prices are only related to net demand for starter homes, intermediate home prices are both related to net demand for starter and net demand for intermediate homes, ND_m. Stated differently, through up-trading the market segment price on the second step of the housing ladder is also affected by market conditions on the first step of the ladder. While an increase in the net demand for intermediate homes has a *direct (*positive) impact on the price of intermediate homes, an increase in the net demand for starter homes has an *indirect (*positive) effect on the price of intermediate homes through equity induced up-trading, e_{ms}.

Family homes

Again, using the market segment equilibrium, the demand for family homes (5) and the price of intermediate homes (9), the price on family homes equals:

10) $$P_f = \frac{1}{p_f}\left[ND_f + \frac{e_{fm}}{p_m}\left[ND_m + \frac{e_{ms}}{p_s}\left[ND_s\right]\right]\right]$$

ND_f is net demand for family homes. The market segment price is again affected *directly* by market conditions in the family home segment and *indirectly* through equity induced up-trading from *both* the starter home and the intermediate home segment. While the price on family homes also is affected by the net demand for starter homes, this indirect effect has to *climb* two steps on the ladder before impacting family home prices and is also conditional on the existence of equity induced up-trading between the segments for intermediate and family homes.

We now turn to comparing the price effects of our heterogeneous housing market structure to those one would experience in a homogenous housing market structure that ignores the interplay between segments. To highlight the value added of a heterogeneous market structure we consider the price effects of shocks to demand equal across market segments. As all segments are exposed to the same shock, the only distinction to a homogenous market structure is the interplay between segments driven by equity induced up-trading. We express shocks in terms of net demand, which for now represents a simplification. The net demand approach will however provide value added when the supply side is endogenised in section 4. We introduce the following:

DEFINITION 1: A shock to net demand *equal across* market segments:
$\delta ND_S = \delta ND_m = \delta ND_f = \delta\overline{ND}$.

A positive (negative) shock to net demand is either due to a reduction (increase) in supply or an increase (decrease) in gross demand. Combining Definition 1 with the expressions (8)-(10) we see price effects differ between market segments along the lines of Poterba (1991) even when shocks to net demand are equal across market segments. While the impact on starter home prices equals $\frac{\delta P_s}{\delta\overline{ND}} = \frac{1}{p_s}$ the impact on intermediate home prices equals $\frac{\delta P_m}{\delta\overline{ND}} = \frac{1}{p_m}\left[1+\frac{e_{ms}}{p_s}\right]$, and that of family home prices, $\frac{\delta P_f}{\delta\overline{ND}} = \frac{1}{p_f}\left[\left[1+\frac{e_{fm}}{p_m}\right]\left[1+\frac{e_{ms}}{p_s}\right]\right]$. We see price effects differ between segments even when exposed to equal shocks. The different price effects are due to the indirect effects accompanying up-trading.

To highlight the effects of equity induced up trading we suppress differences in price elasticities between market segments:[4]

DEFINITION 2: Equality (and unity) between the elasticity of demand in all market segments: $p_S = p_m = p_f = 1$.

This allows us the first of our key results when keeping supply exogenous:

Result 1: *The price effect of a net demand shock equal across market segments is stronger the further up the ladder the segment is located.*

11) $$\frac{\delta P_f}{\delta ND} > \frac{\delta P_m}{\delta ND} > \frac{\delta P_s}{\delta ND} \quad \text{iff} \quad e_{fm} > 0,\ e_{ms} > 0$$

Proof: In Appendix 1.

Result 1 is referred to in the following as *up-trading induced price dispersion*. It shows how shocks to either supply or demand create price dispersion between market segments when housing markets are characterised by equity induced up-trading.

3.2 The house price index, the ladder effect and the housing market multiplier

When inserting for market segment prices, (8)-(10), we find the house price index to equal:

(12) $$P = \frac{\alpha_f}{p_f}\left[ND_f\right] + \left[\frac{\alpha_m}{p_m} + \frac{\alpha_f}{p_f}\left[\frac{e_{fm}}{p_m}\right]\right]\left[ND_m\right] + \left[\frac{\alpha_s}{p_s} + \frac{\alpha_m}{p_m}\left[\frac{e_{ms}}{p_s}\right] + \frac{\alpha_f}{p_f}\left[\left(\frac{e_{fm}}{p_m}\right)\left(\frac{e_{ms}}{p_s}\right)\right]\right]\left[ND_s\right]$$

Expression (12) shows that a market segment impacts the house price index in two distinct ways: Directly through the market segment size *(the size effect)* and indirectly through the segment's position on the housing ladder. The latter is due to equity induced up-trading and is in the following referred to as the *up-trading effect*.

The *size-effect* is determined by the size of a market segment, scaled down by the market segments price elasticity, $\frac{\alpha_i}{p_i}$. The *up-trading effect* is also influenced by the degree of equity

[4] In real housing markets both price and income elasticities in demand differ and are important features for differences in expected wealth effects between housing market segments (Edelstein and Lum, 2002). To highlight the indirect effects these are suppressed. See Borgerrsen (2012) for the role of housing market structure.

induced up-trading, e_{ij}. However, the up-trading effect differs between market segments according to their position on the housing ladder. While absent for family homes, the up-trading effect on the house price index of a shock to net demand for intermediate homes equals $\left[\frac{\alpha_f}{p_f}\left[\frac{e_{fm}}{p_m}\right]\right]$, and the up-trading effect from a from a shock to net demand for starter homes equals $\left[\frac{\alpha_s}{p_s}\left[\frac{e_{ms}}{p_s}\right]+\frac{\alpha_f}{p_f}\left[\frac{e_{fm}}{p_m}\frac{e_{ms}}{p_s}\right]\right]$. While the final step on the ladder only brings with it a direct effect (size effect) to the house price index, segments further down the ladder also bring with them indirect effects (up-trading effects) which are stronger the further down the ladder a segment is located. In the absence of equity induced up-trading, $\mathrm{e}_{\mathrm{fm}} = \mathrm{e}_{\mathrm{ms}} = 0$, the up-trading effects disappears and market segments only impact the house price index through relative sizes.

In order to derive some general assessments on the implications of the up-trading effect for the house price index we consider all market structure segments are equal. This is to neutralize the size effect. We start out by introducing the following:

DEFINITION 3: A *symmetric* housing market structure: $\alpha_S = \alpha_m = \alpha_f = \overline{\alpha}$.

Combining Definitions 1- 3, and using the fact that $\sum_i \overline{\alpha} = 1$ the house price index reduces to:

12a) $$P = \overline{ND} + \overline{\alpha}\overline{ND}\left[e_{ms} + e_{fm}(1+e_{ms})\right]$$

Expression (12) shows how a heterogonous housing market structure introduces an interesting feature into the housing market: The amplification of shocks. A housing market multiplier is characterised by a response in the house price index that exceeds the exogenous shocks to any variable (ε), $\frac{\delta P}{\delta \varepsilon} > 1$. When considering shocks to net demand we find the following:

Result 2: *In the presence of equity induced up-trading a housing market multiplier is present when shocks to net demand are equal between market segments.*

13) $$\frac{\delta P}{\delta \overline{ND}} > 1 \qquad \text{iff} \quad e_{fm} > 0 \text{ or } e_{ms} > 0$$

Proof: Follows directly from (12a).

In the following is Result 2 referred to as the *housing market multiplier*.

The equity gains impacting up-trading links amplify the impact of shocks and introduce a multiplier into the housing market. The multimarket structure provides an explanation for why short-term deviations may come about in a number of equilibrium indicators for the housing market, such as price-construction cost or price-to-income ratios.[5] In fact, even when considering shocks to supply or demand equal across segments shocks are amplified – a feature completely suppressed when considering a homogenous housing market structure.

In addition to amplifying shocks the up-trading effect makes shocks to different segments of the housing ladder result in different features to the house price index.

Maintaining the equality between elasticities as given by Definition 2 and the symmetric housing market structure given by Definition 3, but now allowing for segment-specific shocks and abandoning Definition 1, gives us the following:

Result 3:
When market segments are linked through up-trading a net demand shock results in stronger effects to the house price index the further down the ladder it occurs.

15) $$\frac{\delta P}{\delta ND_f} < \frac{\delta P}{\delta ND_m} < \frac{\delta P}{\delta ND_s} \quad \text{iff} \qquad e_{fm} > 0 \text{ , } e_{ms} > 1 + e_{fm}$$

Proof: In Appendix 1.

Result 3 is referred to in the following as a *complete ladder-effect.*

When housing markets are characterised by equity-induced up-trading, shocks to the starter (intermediate) home segment brings with it stronger effects to the house price index than shocks to intermediate (family) homes. This is due to increased demand for starter (intermediate) homes stimulates equity values in the market segment for starter (intermediate) homes and allows some home owners to trade up to intermediate (family) homes. The cumulative contains both direct and indirect effects making the effects of a shock to the demand for starter homes exceed those of, for instance, family homes, as the latter segment

[5] Borgersen and Sommervoll (2011) show the same reasoning for the price-to-rent ratio in a model including substitution.

does not carry with it additional up-trading, and hence, only direct effects occur to the house price index.

The ladder effect provides a blueprint for an efficient set of housing policies to boost-or dampen-a house price cycle through supply or demand sides measures in different market segments.

3.3 An efficient policy mix

The housing market is affected by macroeconomic, prudential and structural policies both at the supply and demand sides of the market. Hilbers et al. (2008) split housing market policies between:[6]

- Monetary policy (short-term and long-term interest rates and inflationary expectations, directly working through mortgage rates and the cost of borrowing for developers and builders and indirectly through the business cycle)
- Fiscal policy (through direct taxes and subsidies on disposable income and tax deductibility of certain costs and indirectly through the business cycle)
- Structural policy (land and zoning policies affecting construction costs and the supply of housing)
- Supervisory and regulatory policy (through the cost and ease of financial home purchase by affecting capital requirements, loan limits and the legal framework for the use of collateral)

The link between segments that accompany up-trading provides us with policy guidelines at a finer level. First, the complete ladder effect allows statements regarding *when* supply and demand sides measures towards the different market segments are most efficient for a government in need of boosting or dampening house price cycles.

It follows directly from Result 3 that the impact on the house price index of a shock to net demand is stronger the further down the ladder it occurs. Hence, if trying to dampen house price growth by stimulating supply, measures targeting starter homes are more efficient than those addressing intermediate or family homes. Likewise, if the market is in a bust and the government tries to stimulate activity by regulatory interventions increasing mortgage availability, measures with strong impact on mortgage financed housing at the first steps of

[6] See also Angel (2000) for a broad analysis of housing market policy in a global context.

the ladder are most efficient. Knowing how the mortgage structures of young adults are characterised by high Loan-to-value (LTV) and debt-to-income(DTI) ratios, as well as interest only structures (see for instance Leece, 2004), interventions easing (tightening) these restricting are most efficient in kick starting (dampening) the housing market.

In fact, when analysing the housing market in relation to macroeconomic stabilisation, an even more intriguing set of policy combinations can be argued. Consider for example an economy which is in recession and where the housing market is in a bust. The construction industry is labour intensive and an important sector for reducing unemployment (Nickel, 2004). To address a rising unemployment problem the government wants to loosen its land and zoning policy. But at the same time, they are trying to prevent house prices from falling further in order to avoid negative wealth effects on private consumption and financial instability. An increase in employment will boost the economy and *indirectly* impact positively on house prices. The simultaneous increase in housing supply on the other hand will have a *direct* negative effect on house prices. To minimise the direct negative effect on prices the zoning measures should be aimed at expanding the supply of newly built family homes. As this segment carries with it the smallest effect on the house price index, such an intervention maximises the probability that the indirect effect will dominate the direct effect, making the overall impact on house prices positive.

As the multiplier is conditional on equity gains from existing home ownership measures, reducing these gains will dampen imbalances in housing markets. Taxing capital gains or trading fees along the lines of Bruckner (1984) stand in this regard out as fundamental tools.

4. Endogenous supply, net demand and the housing market multiplier

In this section we endogenise housing supply, while maintaining the demand side structure as before. This allows for a complete assessment of how up-trading impacts house prices. Along the lines of Swank (2002), housing supply has two components in each market segment. First, there are newly built houses, N_i, in each segment. Second, we take into account the impact of up-trading on housing supply. Households climbing the housing ladder and demanding houses one step up, are assumed to put their existing home on the market, and simultaneously act both as a buyer and as a seller. Hence, the supply of small houses, S_S, equals newly built small homes, N_S, plus the demand for intermediate homes, D_M. Likewise, the supply of intermediate homes, S_M, equals newly built intermediate homes, N_M, plus the demand for family homes, D_H. At the final step of the ladder where no further up-trading is allowed the supply of family homes, S_F, equals the amount of newly built family homes, N_F.

(15) $$S_S = N_S + D_M$$

(16) $$S_M = N_M + D_F$$

(17) $$S_F = N_F$$

In each market segment newly built houses are functions of the respective market segment price, P_i.

(18) $$N_i = \theta_i + \beta_i P_i$$

where θ_i is an exogenous parameter related to the availability of land for new building in market segment i, and β_i is the supply elasticity. When supply is endogenised the model allows for coexistence of both newly built houses and used homes in each segment. We assume no depreciation of the existing housing stock.[7]

4.1 Market segment prices

When supply is endogenised we differentiate between gross (GD) and net demand (ND) to distinguish between features residing on the supply and demand sides of housing markets, respectively.

We highlight the implications of up-trading and the importance of market segments' different positions on the housing ladder by addressing three questions: First, *how are market segment prices affected by shocks to supply and demand equal across market segments*? Second, *how are market segment prices affected by balanced shocks to net demand in various market segments?* And third, *how are market segment prices affected by balanced shocks to net demand equal across market segments?*

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[7] Allowing for depreciation of the existing housing stock, either due to social or physical reasons, might dampen the supply side effects of up-trading highlighted in this section. On the other hand, upgrading or decoration of the existing stock might counteract the impact of depreciation. For the relation between house prices and upgrading, see for instance, Bajari (2005). To highlight the supply side feature of up-trading we abstract away from depreciation altogether. However, a straightforward interpretation of the model including depreciation is to assume the exogenous supply component θ_i as net of depreciation.

The first question is related to whether the distinctions between a heterogeneous and a homogenous housing market structure carries over from an exogenous to an endogenous supply side where we take into account that nth time buyers also are sellers. When supply is endogenised both newly built and used homes are available for purchase in each segment. Increased demand in one of the segments goes together with increased supply in another.[8] This dual role of demand distinguishes the price effects of shocks to demand from those of shocks to supply. This distinction might modify the up-trading induced price dispersion, the ladder effect and the multiplier derived earlier.

The second and the third questions take into account that as the price effects following shocks to supply and demand not are symmetric, shocks to net demand are more fundamental than in the fixed supply side version of the model. While earlier mainly used for expositional purposes, shocks to net demand might now bring value added to the understanding of housing markets. Net demand is defined in terms of each segment's exogenous supply and demand components, that is, the difference between (gross) demand and newly built homes, $ND_i = GD_i - \theta_i$. As nth time buyers also are sellers housing availability equals the total of newly built and used homes put out for sale in each segment. Being endogenous to the model our expression for net demand does not capture the latter of these two effects. This dual role of demand makes housing markets characterised by features along the lines of The Balanced-Budget Multiplier of Haavelmo. As a shock to demand in one segment simultaneously impacts supply in another segment, a balanced shock to net demand might impact house prices.[9] In relation to the endogenous supply side version of the model we introduce the following definitions:

DEFINITION 4: A shock to demand *equal* across market segments:
$\delta GD_S = \delta GD_m = \delta GD_f = \delta \overline{G}\overline{D}$.

DEFINITION 5: A shock to supply *equal* across market segments:
$\delta\theta_S = \delta\theta_m = \delta\theta_f = \delta\overline{\theta}$.

[8] The price effect of changes in the number of newly built houses in each segment will be parallel to the effect of changes in housing supply in section 3.

[9] The Haavelmo theorem refers to the income effects (concerning the primary impulse) of budget-balance-neutral fiscal policy, where an increase of public expenditures fully financed by a tax increase still induces positive impacts on domestic production.

DEFINITION 6: A *balanced* increase in net demand in market segment i*:*

$\delta GD_i = \delta\theta_i = \delta\vartheta_i \qquad \Rightarrow \delta \mathrm{ND_i} = 0$

DEFINITION 7: A *balanced* increase in net demand *equal across* market segments:

$\delta GD_i = \delta\theta_i = \delta\overline{\vartheta} \ \ \forall i \ \ \Rightarrow \delta \mathrm{ND_i} = 0 \ \ \forall \mathrm{i}$.

When using the equilibrium conditions in (6), and inserting for supply (15-18) and demand (1-5) respectively, we find market segment prices to equal:

(19) $$P_s = \frac{1}{\Omega}\left[\zeta_f \psi_m ND_s - (\zeta_f \beta_m + e_{fm})GD_m - p_m \beta_f \theta_m - p_m \beta_f GD_f - \omega_f \theta_f\right]$$

(20) $$P_m = \frac{1}{\Omega}\left[e_{ms}\zeta_f ND_s + \zeta_s \zeta_f GD_m - \psi_s \theta_m - e_{ms}\zeta_s \beta_f GD_f - \psi_s p_f \theta_f\right]$$

(21) $$P_f = \frac{1}{\Omega}\left[e_{ms} e_{fm} ND_s + \zeta_s e_{fm} GD_m - \psi_s e_{fm} \theta_m + \chi GD_f - \tau\theta_f\right]$$

Compared to the fixed supply side version of the model, ND_i is characterized by θ_i replacing S_i, while gross demand is determined by income and mortgage availability among the relevant household group, $GD_i = c_i + y_i Y_i + m_i M_i$.[10]

4.2 Shocks and dispersion in market segment prices

To highlight the value added a heterogeneous market structure brings to the understanding of housing markets compared to a homogenous market structure, we consider the price effects of shocks to supply and demand given by Definitions 4-7.

From the expressions for market segment prices we see that partial supply side shocks impact negatively on all market segment prices, irrespective of in which segment the shock occurs.

The effects of partial demand shocks are non-conventional: A partial shock to a market segment's (gross) demand has a conventional positive impact on the segment's own price, as well as on all market segment prices further *up* the ladder. The impact on prices further *down* the ladder is negative. This asymmetric price response is due to that households trading up the ladder simultaneously act as buyers and sellers, allowing demand a dual role in the segments

[10] See Appendix 2 for description of parameters and the partial observations from the flexible supply side version of the model.

further down the ladder. Increased demand for intermediate (family) homes implies a simultaneous increase in the availability of starter (intermediate) homes. This increased availability of used homes impacts negatively on prices in the segment where households currently are owners. When considering the price of a starter home, it is negatively related to the demand for intermediate (family) homes through increased availability of starter (intermediate) homes. The price of intermediate homes is positively related both to the demand for starter and to the demand for intermediate homes, while the demand for family homes has a negative impact. As the final step on the ladder, where there are no further steps to climb and no outlet for built up equity, family home prices are positively related to demand shocks irrespective of the segments in which they occur.

Table 1: Shocks to gross demand and market segment prices

Demand shifters	**Starter home prices**	**Intermediate home prices**	**Family home prices**
δGD_s	+	+	+
δGD_m	-	+	+
δGD_f	-	-	+
$\delta GD_S = \delta GD_m = \delta GD_f = \delta \overline{GD}$	+	?	+

When considering a shock to demand equal across market segments as in Definition 4 – and described by Observations 3, 6 and 9 in Appendix 2 – we see both starter and family home prices characterised by conventional positive impacts. For intermediate homes, a segment simultaneously exposed to entry and exit, the impact is related to the relation between the indirect effects residing on the step below (a demand side effect) and on the step above (a supply side effect). The impact on intermediate home prices is in general ambiguous, but positive if there is a relative increase in the demand for intermediate homes.

As all partial supply side shocks are negative, the price effect accompanying a shock given by Definition 5 is naturally also negative in all market segments, as shown by Observations 5, 8 and 11 in Appendix 2.

When considering shocks to supply and demand equal across market segments as given by Definitions 4 and 5 respectively, *up trading induced price dispersion* might carry through to a model with an endogenous supply side. Both the condition for demand and the condition for supply side driven price dispersion are contingent on equity induced up trading generating a relative increase in the demand for intermediate homes.[11]

Result 4: *Both when considering a shock as in Definition 4 (a demand side shock) as well as a shock as in Definition 5 (a supply side shock) is the effect on market segment prices stronger the further up the ladder the segment is located:*

(22) $$\frac{\delta P_f}{\delta G\overline{D}} > \frac{\delta P_m}{\delta G\overline{D}} > \frac{\delta P_s}{\delta G\overline{D}} \quad \text{iff} \quad e_{ms} > 0 \text{ and } e_{fm} > \bar{e}_{fm}.$$

and

(23) $$\left|\frac{\delta P_f}{\delta\bar{\theta}}\right| > \left|\frac{\delta P_m}{\delta\bar{\theta}}\right| > \left|\frac{\delta P_s}{\delta\bar{\theta}}\right| \quad \text{iff} \quad e_{ms} > \tilde{e}_{ms} \text{ and } e_{fm} > \hat{e}_{fm}.$$

Proof: In the appendix.

When turning to the net demand shocks given by Definition 6 and Definition 7 respectively, the results are summarised in Table 2.

Table 2: Shocks to net demand and market segment prices

Demand shifters	Starter home Prices	Intermediate home prices	Family home Prices
$\delta\vartheta_s$	0	0	0
$\delta\vartheta_m$	-	?	?
$\delta\vartheta_f$	-	-	?
$\delta\vartheta_S = \delta\vartheta_m = \delta\vartheta_f = \delta\bar{\vartheta}$	-	?	?

A shock to the net demand for starter homes given by Definition 6 will neither have a *direct* impact on the market segment's own price nor an *indirect* impact on any of the other market segment prices. As the demand side only consists of first time buyers (exogenous) shocks to supply are completely offset by equal (exogenous) shocks to demand.

[11] When using Definition 4 and Observations 4, 7 and 10 in Appendix 2 we may state the first half of Result 4 - the endogenous supply side analogue to Result 1. The latter half of Result 4 – where price dispersion is expressed in absolute terms – comes about when using Definition 5 and Observations 5, 8 and 11 in Appendix 2.

In the segments where repeat buyers operate the dual role of demand comes into play. Through the increased availability of starter homes that accompany households trading up from starter to intermediate homes a balanced increase in the net demand for intermediate homes impacts negatively on starter home prices. The impact on the other market segment prices is ambiguous. For intermediate home prices the positive impact from increased demand is accompanied by two distinct channels impacting negatively on prices: There is both a *direct* (negative) impact through the increased supply of intermediate homes as well as an *indirect* impact through reduced up-trading from below as starter home prices fall and reduces equity values. Through the same features, direct and indirect, there is an ambiguous impact on family home prices, where the latter has to climb two steps on the ladder. For the total impact on intermediate home prices to be positive, there is an upper bound for the *indirect* effect between starter and intermediate homes. For family home prices to rise there is a lower bound on the same indirect effect. The critical levels on the indirect effects between starter and intermediate homes are determined by market elasticities and necessary to ensure a relative increase in the demand for intermediate homes.

A balanced increase in the net demand for family homes impacts negatively on both starter and intermediate home prices through increased availability of used homes. The effect on family home prices is ambiguous, but positive if the indirect effect between starter and intermediate homes is below a critical level.

Finally, we consider the price effects of a balanced increase in net demand equal across market segments as in Definition 7. Using Observations 6, 9 and 12 in Appendix 2 we find important distinctions between segments dominated by first-time and repeat buyers:[12] First, we see how a balanced increase in net demand equal across market segments might impact all market segment prices. This result is completely suppressed when assuming a homogenous housing market structure.

Second, the impact might differ between segments. While unambiguously negative for starter homes due to that the paper abstracts away from housing depreciation, the impact is

[12] The lack of outlet for built up equity impacts the result for family homes. Allowing for bequests, and an outlet for equity gains, could modify this result. In fact, bequests could impact the up-trading induced price dispersion as equity gains originating in the family home segment in part would be fed back into the starter home segment, generating additional indirect effects. On the other hand, bequests could strengthen the multiplier as the indirect effects would spread through the housing ladder in second and third round effects.

ambiguous both for intermediate and for family home prices. The dual role of demand relates to these two latter price effects to the degree of equity induced up-trading between starter and intermediate homes. For intermediate (family) home prices to fall there is a lower (upper) limit for e_{ms}, both necessary to ensure a relative increase in the relevant market segment demand.

4.2 The house price index

When inserting for the market segment prices (19-21) the house price index equals:

$$P = \frac{1}{\Omega}\left(Z_s\left(GD_s - \theta_s\right) + Z_m GD_m - Z_m^{'}\theta_m + Z_f GD_f - Z_f^{'}\theta_f\right) \tag{24}$$

First, we acknowledge the negative relation between the house price index and the supply of both intermediate and family homes (as $Z_i^{'} > 0$, $\mathrm{i = m, f.}$). And, as $Z_s > 0$, the index is negatively (positively) related to the supply of (demand for) starter homes.

When considering the demand for intermediate and family homes respectively, the dual role of demand comes into play (Z_i is indeterminate for i = m, f) . First, for a positive relation between the house price index and the demand for intermediate homes, i.e. $Z_m > 0$, it is necessary for the positive *direct* effect from increased demand for intermediate homes and the positive *indirect* effect from increased demand for family homes following up-trading to exceed the accompanying negative effects from increased availability of both starter and intermediate homes, i.e. $\alpha_s\zeta_f\left(\beta_m + e_{fm}\right) < \zeta_s\left(\alpha_m\zeta_f + \alpha_f e_{fm}\right)$.

For a positive relation between the index and the demand for family homes, i.e. $Z_f > 0$, the positive *direct* effect from the increased demand for family homes must exceed the negative *indirect* effect from increased availability of both starter and intermediate homes that accompany up trading, i.e. $\left(\alpha_s p_m\beta_m + \alpha_m e_{ms}\zeta_s\beta_f\right) < \alpha_f\left(\zeta_s\zeta_f + p_m e_{ms}\right)$.

Turning to the main results from the fixed supply side version of the model we apply Definition 3 and start off with the *complete ladder* analogue:

Result 5: *A shock to demand brings with it stronger effects to the house price index the further down the ladder the shock occurs*

(25) $$\frac{\delta P}{\delta GD_f} < \frac{\delta P}{\delta GD_m} < \frac{\delta P}{\delta GD_s} \qquad \text{iff} \quad \dddot{e}_{ms} < e_{ms} < \ddot{e}_{ms}.$$

Where $\dddot{e}_{ms} = f(e_{fm})$ and $f'(\cdot) < 0$ iff $(\zeta_s - \zeta_f) < 0$. Iff $(\zeta_s - \zeta_f) > 0$ we find $f'(\cdot)$ to be indeterminate. We find $\ddot{e}_{ms} = h(e_{fm})$ where $h'(\cdot) > 0$ iff $(\zeta_s - \zeta_f) > 0$.

Proof: In the appendix.[13]

Second, we turn to the amplification of shocks:

Result 6: *In the presence of equity induced up-trading a housing market multiplier is present when shocks to demand are equal across market segments:*

(26) $$\frac{\delta P}{\delta \overline{\overline{GD}}} > 1 \qquad \text{iff} \quad e_{ms} > e_{ms}'''$$

Where $e_{ms}''' = e(e_{fm})$ and $e'(\cdot) < 0$.

Proof: In the appendix

Even when taking the supply side aspect of up trading into account the housing market might be characterised by both *ladders* and *multipliers*. Both features are conditional on the indirect effect between starter and intermediate homes exceeding a critical limit ensuring a relative increase in intermediate homes.

Turning to the shocks given by Definitions 6 and 7 we see results differ between segments due to the distinct implications of first-time and repeat buyers, respectively.

Using (24) and Definition 6 we find that a balanced shock to net demand for starter homes, $\delta GD_s = \delta\theta_s = \delta\vartheta_s$, does not impact the house price index. The impact is, however, ambiguous, both when considering balanced shocks to the net demand for intermediate homes, $\delta GD_m = \delta\theta_m = \delta\vartheta_m$, as well as when considering balanced shocks to the net demand for family homes, $\delta GD_f = \delta\theta_f = \delta\vartheta_f$. The price effects can be positive or negative depending on parameter values (See the appendix).

[13] See the appendix for a supply side driven ladder effect.

Knowing that balanced shocks to net demand might impact market prices in two segments, we turn to a somewhat intriguing question: Will a balanced shock to net demand equal across market segments as given by Definition 7 impact the house price index?

Result 7: *A balanced shock to net demand equal across market segments has an ambiguous impact on the house price index. The impact is positive (negative) if the equity induced up-trading from the starter home segment is large (small) enough.*

(27) $$\frac{\delta P}{\delta \vartheta} > 0 \text{ iff } e_{ms} < \hat{e}_{ms}. \text{ and } \frac{\delta P}{\delta \vartheta} < 0 \text{ iff } e_{ms} > \hat{e}_{ms}.$$

Where $\hat{e}_{ms} = g(e_{fm})$ and $g'(\cdot) < 0$.

Proof: In the appendix.

In a heterogeneous housing market where the impact of up-trading on both the supply and on the demand side is taken into account, there is a potential impact on the house price index even from balanced shocks to net demand equal across segments. If the demand for intermediate homes increases, even balanced shocks to net demand equal across segments might have a positive impact on the house price index.

5. Conclusions

Taking some key characteristics of housing careers into account this paper delivers value added on the understanding of housing market developments: First that households trading up a housing ladder might induce price dispersion between segments; second that interplay between segments amplifies shocks and introduces a housing market multiplier; and third, how the house price index response is stronger the further down the ladder a shock occurs. The prime focus of this paper is the impact of equity induced up trading. First, we fix supply and highlight the demand side link between segments that accompany equity induced up-trading. The paper shows how shocks impact market segment prices both directly and indirectly, and where the latter effect is the result of equity induced up-trading. The indirect effect makes market segment prices respond more strongly the further up the ladder a segment is located. Up-trading induced price dispersion is observed even if shocks are equal across market segments.

Market segments impact the house price index both directly through their size and indirectly through their position on the housing ladder. The indirect effect also introduces a multiplier into the housing market as equity induced up trading amplifies shocks. The multiplier creates overreactions in house prices compared to developments in household income, mortgage volumes or rent and in the ratios often used to assess whether housing markets are in or out of equilibrium. As the impact on the house price index of a shock to the first step of the housing ladder exceeds those of shocks to the segments further up, policy recommendations at a finer level is allowed when in need of stabilizing house prices: The further down the ladder a policy measure is introduced the stronger is the effect on the house price index.

Second, taking into account that up trading not only links market segments through the demand side, we endogenise supply. Households trading up the ladder simultaneously act as buyers and sellers as existing homes are put out for sale. This allows demand a dual role in housing markets. Endogenising the supply side provides a more differentiated understanding of how various market segments matter for aggregate housing market developments. The model shows how the implications of shocks to market segments exposed to *either* entry or exit differ fundamentally from those of segments simultaneously exposed to *both* entry and exit.

When the impact of up-trading on the supply side of housing markets is included a number of interesting features comes about. First, *up-trading induced price dispersion, the complete ladder effect* and the *housing market multiplier* might still characterize housing markets. Second, the dual role of demand allows net demand shocks non-conventional effects as even balanced shocks to net demand might impact prices.

When taking equity induced up trading into account our heterogeneous market structure brings value added to the understanding of housing markets compared to a homogenous market structure. The idiosyncratic features that come about from the interplay between segments shows how housing markets might carry with them non-conventional features both related to which type of shocks that matter for house prices as well as how the different market segment prices are affected.

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Appendix 1:

Appendix 1 gives the proofs for the results of the fixed supply side version of the model.

Proof of Result 1:

This amounts to taking the derivative of all the market segment prices with respect to $\overline{ND}$ and finding the conditions for, $\frac{\delta P_f}{\delta \overline{ND}} > \frac{\delta P_m}{\delta \overline{ND}} > \frac{\delta P_s}{\delta \overline{ND}}$. Using the partial derivatives of section 3.1, these conditions equal, $\frac{1}{p_f}\left[\left(1+\frac{e_{fm}}{p_m}\right)\left(1+\frac{e_{ms}}{p_s}\right)\right] > \frac{1}{p_m}\left[1+\frac{e_{ms}}{p_s}\right] > \frac{1}{p_s}$. This reduces to

$\left[1+e_{fm}+e_{ms}+e_{fm}e_{ms}\right]>\left[1+e_{ms}\right]>1$, when using Definition 2.For these inequalities to hold it is necessary with indirect effects between all three market segments, i.e. $e_{fm}>0, e_{ms}>0$.

Proof of Result 3:

At issue is the following relation between three partial derivatives, $\frac{\delta P}{\delta ND_f}<\frac{\delta P}{\delta ND_m}<\frac{\delta P}{\delta ND_s}$.

When using Definitions 2 and 3, we find the partial derivatives to equal, $\frac{\delta P}{\delta ND_f}=\overline{\alpha}$,

$\frac{\delta P}{\delta ND_m}=\overline{\alpha}\left(1+e_{fm}\right)$, and $\frac{\delta P}{\delta ND_s}=\overline{\alpha}\left[1+e_{ms}\left(1+e_{fm}\right)\right]$. The relation between the derivatives now reduces to, $\left[1<1+e_{fm}<1+e_{ms}\left(1+e_{fm}\right)\right]$. The necessary conditions for these inequalities to hold are, $e_{fm}>0$, $\mathrm{e}_{\mathrm{ms}}>1+e_{fm}$.

Appendix 2:

Appendix 2 gives the details of the endogenous supply side model described in section 4. Applying the equilibrium condition in (6) and inserting for supply (8-12) and demand (18-21) we derive expressions for both market segment prices and the house price index. The appendix also presents Observations 3–11 which only are referred in the text.

Parameters and simplifying notation:

Gross demand is determined by income and mortgage availability among the relevant market segment's household group, $GD_i=c_i+y_iY_i+m_iM_i$. The cumulative trickle-down effect on family home prices from shocks to the starter home segment is given by, $\omega_f=p_fp_m$. The market segment elasticity, $\zeta_i=\beta_i+p_i$, captures both the market segment's supply and demand side elasticity. The gross market segment elasticity for starter, $\psi_s=\zeta_s+e_{ms}$, and for intermediate homes, $\psi_m=\zeta_m+e_{fm}$, combines the market segment elasticity with the indirect effects in demand. To simplify notation we allow, $\sigma_m=\beta_m+e_{fm}$, to combine the supply elasticity and the indirect effects in demand among intermediate homes and the parameters $\chi=\zeta_s\zeta_f+e_{ms}\beta_m$ and $\tau=\psi_s\psi_m+p_me_{ms}$. The price effects are scaled down by the factor

$\Omega = p_f(\psi_s\zeta_m + p_m e_{ms})$, where $\Omega > 0$ (As $p_f > 0$ and $(\psi_s\zeta_m + p_m e_{ms}) > 0$. The latter reduces to $\chi = (\zeta_s\zeta_m + e_{ms}\beta_m) > 0$).

Starter home prices

(1a) $P_s = \frac{1}{\Omega}\left[\zeta_f\psi_m ND_s + (p_m\zeta_f - \zeta_f\psi_m)GD_m - (p_m\zeta_f)\theta_m + (\omega_f - p_m\zeta_f)GD_f - \omega_f\theta_f\right]$

Shocks to supply impacts negatively on starter home prices irrespective of in which segment the shock occurs. While the (gross) demand for starter homes has conventional positive impact on its own market segment price, both the demand for intermediate and the demand for family homes impacts negatively on starter home prices. This is due to that households trading up the ladder simultaneously act as sellers in the segment where they are currently owners. Increased demand for intermediate homes stimulates the availability of starter homes and hence, impacts negatively on starter home prices,

$\frac{\delta P_s}{\delta GD_m} = (p_m\zeta_f - \zeta_f\mu_m) = -[\zeta_f(\beta_m + e_{fm})] < 0$. Through the increased availability of intermediate homes that accompany increased demand for family homes, starter home prices are affected negatively, as the lower price on intermediate homes allows more young adults to trade up to intermediate homes and increase the supply of starter homes,

$\frac{\delta P_s}{\delta GD_f} = (\omega_f - p_m\zeta_f) = -(p_m\beta_m) < 0.$

Turning to the price effects of shocks given by Definitions 4, 5, 6 and 7 we find:

Observation 3: *A shock to supply equal across market segments impacts negatively on starter home prices*

(2a) $$\frac{\delta P_s}{\delta\bar{\theta}} = -\left[\frac{1}{\Omega}\left[(\zeta_f(\zeta_m + e_{fm})) + \zeta_f p_m + \omega_f\right]\right] < 0$$

Proof: Follows directly from (1a).

The reasoning above tells us that the dual role of demand comes into play when analysing the impact of a shock given by Definition 5 on starter home prices. However, as the positive effect of increased demand for starter homes dominates the negative effect of increased demand for intermediate and family homes, is the total effect unambiguously positive.

Observation 4: *A shock to demand equal across market segments impacts positively on starter home prices*

(3a)
$$\frac{\delta P_s}{\delta \overline{GD}} = \frac{1}{\Omega}\left[\omega_f\right] > 0 .$$

Proof: Follows directly from (1a).

It also follows directly from (1a) that a balanced increase in the net demand for starter homes (as in Definition 6), $\delta GD_S = \delta\theta_S$, does not impact on the price of starter homes, $\frac{\delta P_s}{\delta \vartheta_s} = 0$. A balanced increase in the net demand for intermediate homes on the other hand, $\delta GD_m = \delta\theta_m$, has a negative impact, $\frac{\delta P_s}{\delta \vartheta_m} = -\frac{1}{\Omega}\left[\zeta_f\left(\zeta_m + e_{fm}\right)\right] < 0.$ Likewise a balanced increase in the net demand for intermediate homes, $\delta GD_f = \delta\theta_f$, is also negative, $\frac{\delta P_s}{\delta \vartheta_f} = -\frac{1}{\Omega}\left[\zeta_f p_m\right] < 0.$

The intuition is straightforward: Increased supply of intermediate (family) homes has a direct negative impact on starter (intermediate) home prices. In addition, increased demand for intermediate (family) homes increases the availability of used starter (intermediate) homes through the up-trading process, something that again impacts negatively on starter home prices and makes the total effect unambiguously negative.

Observation 5: *A balanced increase in net demand equal across market segments impacts negatively on starter home prices:*

4a)
$$\frac{\delta P_s}{\delta \vartheta} = -\left[\frac{1}{\Omega}\left(\zeta_f(\zeta_m + e_{fm} + p_m\right)\right] < 0$$

Proof: Follows directly from the reasoning above as all partial effects are zero or negative.

Intermediate home prices

5a)
$$P_m = \frac{1}{\Omega}\left[e_{ms}\zeta_f ND_s + \zeta_s\zeta_f GD_m - \psi_s\theta_m - e_{ms}\zeta_s\beta_f GD_f - \psi_s p_f\theta_f\right]$$

Supply shocks impacts negatively on the price of intermediate homes irrespective of in which segments the shock occurs. When considering demand shocks, the price effect is contingent on the segments position on the housing ladder. While positively related to both the demand for starter and to the demand for intermediate homes, prices of intermediate homes are negatively related to the demand for family homes. Households trading up from intermediate to family homes simultaneously act as sellers of intermediate homes, which increased availability impacts negatively on prices.

The price effects following shocks given by the Definitions 5 and 6 are:

Observation 6: *A shock to supply equal across market segments impacts negatively on intermediate home prices*

6a) $$\frac{\delta P_m}{\delta\bar{\bar{\theta}}} = -\left[\frac{1}{\Omega}\left(e_{ms}\zeta_f - \left(e_{ms} + \zeta_s\right) + \left(e_{ms} + \zeta_s\right)p_f\right)\right] < 0$$

Proof: Follows directly from (5a).

Observation 7: *A shock to demand equal across market segments has an ambiguous impact on intermediate home prices*

7a) $$\frac{\delta P_m}{\delta\bar{\bar{GD}}} = \frac{1}{\Omega}\left[e_{ms}\zeta_{\mathrm{f}} + \zeta_{\mathrm{s}}\zeta_{\mathrm{f}} - \mathrm{e}_{\mathrm{ms}}\zeta_s\beta_{\mathrm{f}}\ \right]$$

The dual role of demand makes the total impact of a shock given by Definition 4 ambiguous. Knowing that, $\Omega > 0$, we find the necessary condition for, $\frac{\delta P_m}{\delta\bar{\bar{GD}}} > 0$, by rearranging $\left[e_{ms}\zeta_{\mathrm{f}} + \zeta_{\mathrm{s}}\zeta_{\mathrm{f}} - \mathrm{e}_{\mathrm{ms}}\zeta_s\beta_{\mathrm{f}}\ \right]$. Expressed in terms of the indirect effects arising from starter homes the condition for, $\frac{\delta P_m}{\delta\bar{\bar{GD}}} > 0$, can be expressed as, $e_{ms} < \frac{\zeta_s}{\left(\zeta_s/\zeta_f\right)\beta_f - 1}$. For intermediate home prices to rise is it necessary that the indirect effect from the starter home segment is small enough to ensure a relative increase in the demand for intermediate homes.

When turning to net demand, the price effects of shocks given by Definition 6 can be reasoned as follows: It follows directly from (5a) that a balanced increase in net demand for starter homes, $\delta\mathrm{GD}_{\mathrm{s}} = \delta\theta_{\mathrm{S}} = \delta\vartheta_{\mathrm{s}}$, does not impact on intermediate home prices, $\frac{\delta\mathrm{P}_{\mathrm{m}}}{\delta\vartheta_{\mathrm{s}}} = 0$. For a balanced increase in net demand for intermediate homes, $\delta\mathrm{GD}_{\mathrm{m}} = \delta\theta_{\mathrm{m}} = \delta\vartheta_{\mathrm{m}}$, we find, $\frac{\delta P_m}{\delta\vartheta_m} = \frac{1}{\Omega}\left(\zeta_s\left(\zeta_f - 1\right) - e_{ms}\right)$, which in general is ambiguous. The condition for, $\frac{\delta P_m}{\delta\vartheta_m} > 0$, is again, $\left(e_{ms} < \zeta_s\left(\zeta_f - 1\right)\right)$, restricting the size of the indirect effect between starter and intermediate homes. Also when considering the net demand for family homes,

$\delta GD_f = \delta\theta_f = \delta\vartheta_f$, the impact is ambiguous, $\frac{\delta P_m}{\delta \vartheta_f} = \frac{1}{\Omega}\left(e_{ms}\zeta_s\beta_f - \psi_s p_f\right)$. After some rearranging the condition, $\frac{\delta P_m}{\delta \vartheta_f} > 0$, can be expressed as, $e_{ms} > \frac{\zeta_s p_f}{\zeta_s \beta_f - p_f}$.

Applying Definition 7 and the reasoning above we may state:

Observation 8: *A balanced increase in net demand equal across market segments has an ambiguous impact on intermediate home prices*

8a) $$\frac{\delta P_m}{\delta \vartheta} = \frac{1}{\Omega}\left[\left(\zeta_s\left(\beta_f - 1\right)\left(1 + e_{ms}\right)\right) - e_{ms} p_f\right]$$

Proof: Follows directly from the reasoning above.

For intermediate home prices to rise, $\frac{\delta P_m}{\delta \vartheta} > 0$, the condition is, $e_{ms} < \frac{\zeta_s\left(\beta_f - 1\right)}{\left[1 + p_f + \beta_s + p_s\right]}$. As long as the indirect effects between starter and intermediate homes ensure a relative increase in the demand for intermediate homes will a balanced increase in net demand equal across market segments have a positive impact on intermediate home prices.

<u>Family home prices</u>

9a) $$P_f = \frac{1}{\Omega}\left[e_{ms} e_{fm} ND_s + \zeta_s e_{fm} GD_m - \psi_s e_{fm}\theta_m + \chi GD_f - \tau\theta_f\right]$$

When it comes to family home prices, the relation to both supply and demand is conventional. We may then state the price effect of shocks given by Definition 4 and Definition 5 directly:

Observation 9: *A shock to supply equal across market segments impacts negatively on family home prices.*

10a) $$\frac{\delta P_f}{\delta \bar{\theta}} = -\left[\frac{1}{\Omega}\left(\psi_s\left(\psi_m + e_{fm}\right) + e_{\mathrm{ms}}\left(p_{\mathrm{m}} + e_{\mathrm{fm}}\right)\right)\right] < 0$$

Proof: Follows directly from (9a) and the definition of τ.

Observation 10: *A shock to demand equal across market segments impacts positively on family home prices.*

10b) $$\frac{\delta P_f}{\delta \overline{GD}} = \left[\frac{1}{\Omega}\left(e_{ms} e_{fm} + \varepsilon_s e_{\mathrm{fm}} + \chi\right)\right] > 0$$

Proof: Follows directly from (9a) and the definition of χ.

As with both starter and intermediate home prices a balanced increase in net demand for starter homes, $\delta GD_s = \delta\theta_S = \delta\vartheta_s$, has no impact on family home prices, $\frac{\delta P_f}{\delta\vartheta_s} = 0$. When considering a balanced increase in the net demand for intermediate homes, $\delta GD_m = \delta\theta_m = \delta\vartheta_m$, the impact is negative, $\frac{\delta P_f}{\delta\vartheta_m} = e_{fm}(\zeta_s - \psi_s) < 0, \text{ iff } e_{ms}, e_{fm} > 0$. When considering a balanced increase in the net demand for family homes, $\delta GD_f = \delta\theta_f = \delta\vartheta_f$, the impact is in general ambiguous, $\frac{\delta P_f}{\delta\vartheta_f} = \zeta_s(\zeta_f - \zeta_m - e_{fm}) - e_{ms}(\beta_m + p_m - \zeta_m - e_{fm})$. The condition for family home prices to increase, $\frac{\delta P_f}{\delta\vartheta_f} > 0$ introduces an upper bound for the indirect effect between starter and intermediate homes, $e_{ms} < \frac{\zeta_s(\zeta_f - \zeta_m - e_{fm})}{e_{fm}}$.

Turning to a shock given by Definition 7 we find that:

Observation 11: *A balanced increase in net demand equal across market segments has an ambiguous impact on family home prices*

$$\frac{\delta P_f}{\delta\overline{\overline{\vartheta}}} = \frac{1}{\Omega}\left[\gamma_s(\beta_m - 2e_{fm} - p_m - \zeta_m) + \zeta_s(\zeta_f - \zeta_m - e_{fm})\right] \tag{11}$$

Proof: Follows from (9a).

The condition for, $\frac{\delta P_f}{\delta\overline{\overline{\vartheta}}} > 0$ is $e_{ms} > \frac{\zeta_s(\zeta_m + e_{fm} - \zeta_f)}{\beta_m - 2e_{fm} - p_m - \zeta_m}$, to ensure that the increase in the demand for intermediate homes is large enough to stimulate the demand for family homes.

Proof of Result 4:

We want to find the conditions for *up trading induced price dispersion* when shocks reside on the demand side of the housing market. When applying Definition 4 we find the condition to equal, $\frac{\delta P_f}{\delta\overline{\overline{GD}}} > \frac{\delta P_m}{\delta\overline{\overline{GD}}} > \frac{\delta P_s}{\delta\overline{\overline{GD}}}$. We start out by considering, $\frac{\delta P_m}{\delta\overline{\overline{GD}}} > \frac{\delta P_s}{\delta\overline{\overline{GD}}}$. This inequality can be expressed as $\zeta_f(\zeta_s + e_{ms}) - e_{ms}\zeta_s\beta_f > p_m p_f$ which again can be rearranged as,

$e_{ms} > \dfrac{p_m p_f - \zeta_f \zeta_s}{\zeta_f - \zeta_s \beta_f}$. For $e_{ms} > 0$ to hold, it is necessary that $\left(p_m p_f > \zeta_s \zeta_f\right)$ and $\left(\zeta_f > \zeta_s \beta_f\right)$(Alternatively that $\left(p_m p_f < \zeta_s \zeta_f\right)$ and $\left(\zeta_f < \zeta_s \beta_f\right)$).

The condition, $\dfrac{\delta P_f}{\delta \overline{GD}} > \dfrac{\delta P_m}{\delta \overline{GD}}$, equals,
$e_{fm}\left(e_{ms} + \zeta_s\right) + \zeta_s \zeta_f + e_{ms}\beta_m > e_{ms}\zeta_f + \zeta_s \zeta_f - e_{ms}\zeta_s \beta_f$. This inequality is easily rearranged as $e_{ms} < \dfrac{\zeta_s e_{fm}}{\zeta_f - \zeta_s \beta_f - \left(e_{fm} + \beta_m\right)} = \vec{e}_{ms}$. We easily see how $\vec{e}_{ms} = i(e_{fm})$ and $i'(\cdot) > 0$.

When combining the two inequalities we find the combined constraint as,
$\vec{e}_{ms} \equiv \dfrac{p_m p_f - \zeta_f \zeta_s}{\zeta_f - \zeta_s \beta_f} < e_{ms} < \dfrac{\zeta_s e_{fm}}{\zeta_f - \zeta_s \beta_f - \left(e_{fm} + \beta_m\right)} \equiv \vec{e}_{ms}$.

The conditions for *up trading induced price dispersion* in the case of supply side shocks given by Definition 3 are, $\left|\dfrac{\delta P_f}{\delta \overline{\theta}}\right| > \left|\dfrac{\delta P_m}{\delta \overline{\theta}}\right| > \left|\dfrac{\delta P_s}{\delta \overline{\theta}}\right|$. We start out by considering, $\left|\dfrac{\delta P_m}{\delta \overline{\theta}}\right| > \left|\dfrac{\delta P_s}{\delta \overline{\theta}}\right|$, which equals $e_{ms}\zeta_f + \left(e_{ms} + \zeta_s\right) + \left(e_{ms} + \zeta_s\right)p_f > \zeta_f\left(\zeta_m + e_{fm}\right) + p_m\left(\zeta_t + p_f\right)$.
After some rearranging, we rewrite this inequality as
$e_{ms} > \dfrac{\zeta_f\left(\zeta_m + e_{fm}\right) + p_m(\zeta_f + p_f) - \zeta_s\left(1 + p_f\right)}{\zeta_f + p_f + 1} = \tilde{e}_{ms}$.
Likewise, the condition, $\left|\dfrac{dP_f}{d\overline{\theta}}\right| > \left|\dfrac{\delta P_m}{\delta \overline{\theta}}\right|$, can be written as
$\psi_s\left(\psi_m + e_{fm}\right) + e_{ms}\left(p_m + e_{fm}\right) > e_{ms}\zeta_f + \left(1 + p_f\right)\left(e_{ms} + \zeta_s\right)$. This is rearranged as
$e_{fm} > \dfrac{\zeta_s\left(1 + p_f - \zeta_f\right) + \zeta_f + (1 + p_f) - p_m - \zeta_f}{3e_{ms} + 2\zeta_s} \equiv \hat{e}_{fm}$.
We see how $\tilde{e}_{ms} = \pi\left(e_{fm}\right)$ where $\pi'(\cdot) > 0.$ and $\hat{e}_{fm} = \gamma\left(e_{ms}\right)$ where $\gamma' < 0.$

The house price index

When inserting for the market segment prices (21- 23) the house price index equals:

24) $$P = \frac{1}{\Omega}\left(Z_s\left(GD_s - \theta_s\right) + Z_m GD_m - Z_m^{'}\theta_m + Z_f GD_f - Z_f^{'}\theta_f\right)$$

where

$$Z_s = \left(\alpha_f e_{ms} e_{fm} + \alpha_m e_{ms}\zeta_f + \alpha_s \zeta_f \psi_m\right)$$

$$Z_m = \left(\alpha_f e_{fm}\zeta_s + \alpha_m \zeta_s \zeta_f - \alpha_s \zeta_f\left(\beta_m + e_{fm}\right)\right)$$

$$Z'_m = \left(\alpha_f e_{fm}\psi_s + \alpha_m \psi_s + \alpha_s p_m \zeta_f\right)$$

$$Z_f = \left(\alpha_f(\zeta_s\zeta_f + p_m e_{ms})) - \alpha_m e_{ms}\zeta_s\beta_f - \alpha_s p_m \beta_m\right)$$

$$Z'_f = \left(\alpha_f \tau + \alpha_m \psi_s p_f + \alpha_s \omega_f\right)$$

Proof of Result 5:

The complete ladder effect in the case of demand side shocks, $\frac{\delta P}{\delta GD_s} > \frac{\delta P}{\delta GD_m} > \frac{\delta P}{\delta GD_f}$, amounts to $Z_s > Z_m > Z_f$.

We start out by considering the inequality $Z_s > Z_m$, which by using Definition 3 equals $e_{fm}e_{ms} + e_{ms}\zeta_f + \zeta_f\left(\zeta_m + e_{fm}\right) > e_{fm}\zeta_s + \zeta_s\zeta_f - \zeta_f\beta_m - \zeta_f e_{fm}$. This inequality can be expressed as $e_{ms} > \frac{\zeta_f\left(\zeta_s - \zeta_m\right) - \zeta_f\beta_m + e_{fm}\left(\zeta_s - \zeta_f\right)}{e_{fm} + \zeta_f} \equiv \dddot{e}_{ms}$

We see how $\dddot{e}_{ms} = f(e_{fm})$ where $f'(\cdot) < 0$ iff $\left(\zeta_s - \zeta_f\right) < 0$. Iff $\left(\zeta_s - \zeta_f\right) > 0$ $f'(\cdot)$ is in general indeterminate.

When considering the inequality $Z_m > Z_f$, and again using Definition 3, we can write this as $e_{fm}\zeta_s + \zeta_s\zeta_f - \zeta_f\beta_m - \zeta_f e_{fm} > \zeta_s\zeta_f + p_m e_{ms} - e_{ms}\zeta_s\beta_f - p_m\beta_m$. After some rearranging, this reduces to $e_{ms} < \frac{\beta_m\left(p_m + \zeta_f\right) + e_{fm}\left(\zeta_s - \zeta_f\right)}{p_m - \zeta_s\beta_f} \equiv \ddot{e}_{ms}$. When assuming $\left(p_m - \zeta_s\beta_f\right) > 0$ we have $\ddot{e}_{ms} > 0$. We find $\ddot{e}_{ms} = h(e_{fm})$ where $h'(\cdot) > 0$ iff $\left(\zeta_s - \zeta_f\right) > 0$.

Combining the two restrictions on the intensity of equity induced up trading between starter and intermediate homes necessary for a complete ladder effect we have $\dddot{e}_{ms} < e_{ms} < \ddot{e}_{ms}$.

The *complete ladder effect* is in the case of supply side shocks, $\frac{\delta P}{\delta\theta_s} > \frac{\delta P}{\delta\theta_m} > \frac{\delta P}{\delta\theta_f}$, derived in terms of absolute values, which conditions equal $\left|Z'_f\right| < \left|Z'_m\right| < \left|Z_s\right|$.

When using Definition 2 the latter inequality reduces to $e_{ms}e_{fm} + e_{ms}\zeta_f + \zeta_f\zeta_m + \zeta_f e_{fm} > e_{fm}\zeta_s + e_{fm}e_{ms} + \zeta_s + e_{ms} + p_m\zeta_{fm}$. After some rearranging this inequality can be expressed as $e_{ms} > \frac{e_{fm}\left(\zeta_s - \zeta_f\right) + \zeta_s + \zeta_f\beta_m}{\zeta_f - 1} = e''_{ms}$. We see how $e''_{ms} = k\left(e_{fm}\right)$ where $k'(\cdot) > 0$ when $\left(\zeta_s - \zeta_f\right) > 0$ and $\left(\zeta_f > 1\right)$.

Turning to $\left|Z_m^{'}\right| > \left|Z_f^{'}\right|$ we express this inequality as

$$e_{fm}\zeta_s + e_{fm}e_{ms} + \zeta_s + e_{ms} + p_m\zeta_{fm} > p_m p_f + \zeta_s p_f + e_{ms}p_f + p_m e_{ms} + \zeta_s\zeta_m + \zeta_s e_{fm} + e_{ms}\zeta_m + e_{ms}e_{fm}$$

By elimination and rearranging this can be reduced to $e_{ms} < \dfrac{\zeta_s\left(1 - p_f - \zeta_m\right) + p_m\beta_f}{\beta_m - p_f - 1} = e'_{ms}$.

When shocks reside on the supply side of housing markets, a complete ladder can be expressed in terms of a critical range for the indirect effect from intermediate to family homes, $e''_{fm} < e_{fm} < e'_{fm}$.

Proof of Result 6:

This existence of a housing market multiplier, $\dfrac{\delta P_m}{\delta GD} > 1$, amounts to finding the conditions for $\dfrac{1}{\Omega}\left(Z_f + Z_M + Z_s\right) > 1$. Where $\Omega = p_f\left(\psi_s\zeta_m + p_m e_{ms}\right)$, and the expressions for Z - when using Definition 3 – equals $Z_s = \left(e_{ms}e_{fm} + e_{ms}\zeta_f + \zeta_f\psi_m\right)$,

$$Z_m = \left(e_{fm}\zeta_s + \zeta_s\zeta_f - \zeta_f\left(\beta_m + e_{fm}\right)\right)$$

$$Z_f = \left(\left(\zeta_s\zeta_f + p_m e_{ms}\right)\right) - e_{ms}\zeta_s\beta_f - p_m\beta_m\right)$$

We find the criteria for a housing market multiplier expressed in terms of the equity induced up trading between starter and intermediate homes after inserting for the simplified expressions for Z_i

$$e_{ms} > \frac{p_m\zeta_s\zeta_f + p_m\beta_m - \zeta_s\zeta_f + \zeta_f\beta_m - \zeta_f\zeta_m - \zeta_s e_{fm}}{\left(\zeta_f + p_m - \zeta_s\beta_f - p_f\left(\zeta_m + p_m\right) + e_{fm}\right)} \equiv e'''_{ms}.$$

Where $e'''_{ms} = e\left(e_{fm}\right)$ and $e^{'}(\cdot) < 0$.

Proof of Result 7:

From (24) we see that a balanced shock to net demand for starter homes does not impact on the house price index.

The impact of a shock to net demand equal across market segments on the house price index is reduced to, $\dfrac{\delta P}{\delta\overline{\vartheta}} = \dfrac{1}{\Omega}\left(Z_m - Z_m^{'} + Z_f - Z_f^{'}\right)$.

Using Definition 3 to supress the differences in segment size, and simplifying, we have

$$Z_m = e_{fm}\left(\zeta_s - \zeta_f\right) + \zeta_f\left(\zeta_s - \beta_m\right), \quad Z_f = e_{ms}p_m - \zeta_s\beta_f - p_m\beta_m + \zeta_s\zeta_f$$

$$Z_m^{'} = \left(\zeta_s + e_{ms}\right)\left(1 + e_{fm}\right) + p_m\zeta_f \quad Z_f^{'} = \left(\zeta_s + e_{ms}\right)\left(\zeta_m + e_{fm}\right) + p_m e_{ms} + \left(\zeta_s + e_{ms}\right)p_f + p_m p_f$$

When inserting, and rearranging, we express the condition for, $\dfrac{\delta P}{\delta\overline{\vartheta}} > 0$

$e_{ms} < \dfrac{\zeta_f(\zeta_s - \beta_m - p_m - e_{fm}) - \zeta_s(\zeta_f - \zeta_m + p_f - e_{fm})}{\zeta_s(1+\beta_f) + \zeta_m + p_f} \equiv \hat{e}_{ms}$. Where $\hat{e}_{ms} = g(e_{fm})$ and $g'(\cdot) < 0$.