

Mortgage Supply, LTV and Risk Pricing

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Abstract:

This paper highlights the contributions from the supply side of mortgage markets to the variations in the LTV-ratio. The paper starts by deriving the optimal LTV-ratio for a profit maximizing mortgagee that supply mortgages using housing as collateral. As the LTV-ratio represents the risk exposure of a mortgagee, the optimal LTV-ratio varies according to moral hazard, risk pricing, funding structure, lending volume and collateral value. Thinking in terms of social welfare, the optimal LTV-ratio is our model one where mortgagees are paid for their risk exposure. Our framework allows us to see how different supply side components create a wedge between the profit maximizing LTV-ratio and the socially optimal LTV-ratio. It also allows for rather straightforward arguments regarding macro-prudential policy. The paper continues by analyzing a mortgage's risk pricing response to falling house prices and an LTV-ratio that exceeds the LTV-ratio at origination. The paper derives a kinked relation between the mortgage rate and the LTV-ratio *ex post*, distinguishing between risk pricing *ex ante* and *ex post*.

Introduction

Why do loan-to-value (LTV)-ratios vary across countries and periods? And, why did so many countries see higher LTV-ratios in the period preceding the financial crisis?

The literature on LTV-ratios is extensive. Some papers see the LTV-ratio as an exogenous variable determined by government regulation. Others allow for endogenous LTV-ratios derived from the interaction between the supply and the demand side of mortgage markets. Demand side factors are obviously important for variations in LTV-ratios. A young population might demand higher LTV-ratios than an older population with higher savings and accumulated equity. In addition, in economies where housing appreciations exceed wage growth is the need to borrow a larger share of the price of a dwelling stimulating the demand for higher LTV-ratios.

However, what is actually the supply side contribution to how LTV-ratios evolve?

The paper analyses the optimal Loan-to-Value (LTV) ratio and the risk pricing strategy of a profit-maximization mortgagee. When considering the expected profit of a mortgagee we distinguish between credit risk assessments *ex ante* and *ex post*, where the former is relevant for determining the LTV-ratio offered at origination, and the latter for addressing the risk pricing response to changes in mortgagees' risk exposure.

The aim of the paper is to bring factors important for mortgagees' optimal LTV-ratio and risk pricing onto a standardized and non-technical playing field. Applying the Furlong and Keeley (1987, 1989) framework the paper highlights the relation between the risk exposure of a mortgagee and the LTV-ratio at origination, as well as the mortgagees' risk pricing response to changing collateral values. Addressing a mortgagee's risk taking both *ex ante* and *ex post* in a simplistic common framework helps us understand both mortgage market structures and how mortgage markets respond to shocks. The framework also provides guidance for reasoning about macro-prudential policy.

Abstracting away from the demand side of mortgage markets, the paper first shows how the optimal LTV-ratio for a profit-maximizing mortgagee facing moral hazard but not attracting deposits, is one where the mortgagee at the margin is paid for its risk exposure. Thinking in terms of social welfare, the same condition will determine the socially optimal LTV-ratio in the absence of demand side considerations. The paper expands the reasoning on LTV-ratios and the risk exposure of a mortgagee by introducing mortgage market features residing on the

supply side of the market. The paper includes external funding to allow for LTV-ratio arguments both related to deposit insurance and capital-adequacy regulations. Focusing the supply side of mortgage markets, the paper also highlights the impact of mortgage volumes and collateral appreciations for the profit maximizing LTV-ratio. The wedge these characteristics create, compared to the socially optimal LTV-ratio, introduces a rationale for macro-prudential features in the model.

The paper continues by analyzing a mortgagees' risk pricing response to a housing depreciation. Considering a case where, due to a fall in house prices, the current LTV-ratio exceeds the LTV-ratio at origination, we turn to a situation where a mortgagee's current risk exposure (*ex post*) exceeds that at origination (*ex ante*). When addressing risk-based pricing the paper shows how a higher LTV-ratio impacts positively on risk pricing through a number of channels, creating a kinked-relation between the LTV-ratio and the mortgage rate *ex post*. This kinked relation is in accordance with risk-based pricing allowing the mortgage rate to be influenced both by the higher probability of default and by the larger expected loss in case of default.

The rest of the paper is structured as follows. In the next section, we relate the model to the prevailing literature. In section three, we present the model. The fourth section relates (sequentially) the LTV-ratio to moral hazard, risk pricing, funding, lending and collateral. The fifth part derives the kinked relation between the LTV-ratio and the interest rate margin (the mortgage rate). The last part concludes.

Related literature and the model set-up

Deriving the LTV-ratio from the supply side of mortgage markets, and seeing it as a measure of a mortgagee's risk exposure, contrasts the approach interpreting the (inverse of the) LTV-ratio as an indicator of how developed a mortgage market is (see for instance Jappelli and Pagano (1989)).

Variations in the LTV-ratio are seen both over time and across markets. Calza et al (2013) reports that LTV-ratios differ from 50 percent in Italy to 112 percent in the Netherlands, while Amior and Halket (2014) show variations in LTV-ratios across US cities. In particular is the latter kind of LTV variation difficult to relate to the degree of mortgage market depth.

Treating the LTV-ratio as the risk exposure of a mortgagee our framework draws on that of Furlong and Keeley (1987, 1989) and Keeley (1990). Taking external funding into account,

the paper shows (conventionally) how declining capital-to-asset ratios and favorable deposit insurance schemes impact positively on a mortgagee's risk exposure and the optimal LTV-ratio. The model also shows how moral hazard and risk pricing impact the LTV-ratio.

The supply side focus allows us to highlight the endogenous credit constraint of Kiyotaki and Moore (1997). While higher collateral values stimulate lending, is there a positive ~~feed-~~
~~back~~feedback effect from lending to house prices and collateral values. Such bidirectional causality between house prices and mortgage volumes is for instance shown by Anundsen and Jansen (2013) or Sophocles and Vlassopoulos (2009). Like Pavlov and Wachter (2011) and Adelino et al (2012) we allow for a positive impact from credit supply to house prices. Our credit supply approximation is the prevailing LTV-ratio, argued by Englund (2011) as a better indicator of credit supply than, for instance, the more commonly used credit volume, which is influenced by both the supply and the demand side of mortgage markets. Kim (2007) relates the LTV-ratio to the price-rent ratio and Duca et al (2011) shows that incorporating the LTV-ratio into the price-to-rent ratio helps to overcome the problem related to that most US house price models breaks-down in the mid-2000. However, Duca et al (2010) is one of the few (?) papers that explicitly link the LTV-ratio to house prices empirically. The paper finds a long-run elasticity of house prices with respect to the LTV-ratio for first-time buyers $\in (0.8, 1.1)$.

Mian and Sufi (2011) show how housing appreciations might stimulate lending both from existing homeowners and, by allowing a 'financially riskier' set of new home-buyers to enter the mortgage market, from first-time entrants. Entry of 'financially riskier' households into owner-occupation is also the focus of Gabriel and Rosenthal (2010). Analyzing the guidelines given by the US congress to Fannie Mae and Freddie Mac to increase funding of low down-payment (and high LTV) mortgages, they see new entry and increased homeownership rates as a main result. Another important finding is how higher LTV-ratios were accompanied by increased lending. In our framework we distinguish between the effects from entry into homeownership and the effects from existing homeowners by relating both collateral values and mortgage volumes to the LTV-ratio. The positive relation between the LTV-ratio and mortgage volume captures the effect of a higher LTV-ratio on entry. Higher lending volumes impact positively on operating income and, as a mortgagor's debt increases, on loss given default. While the former stimulates risk taking and the LTV-ratio, is the impact from the latter negative, making the total effect on the LTV-ratio ambiguous. To highlight the impact from existing homeowners we allow for a positive relation between the LTV-ratio and

collateral. As appreciations reduce losses in case of default, there is a positive relation between the optimal LTV-ratio and collateral values.

Incorporating moral hazard, risk pricing, external funding (and the accompanying regulations) as well as the strategic aspects arising from the impact of mortgage supply on mortgage volumes and collateral values allows the model to bring a number of factors that might cause variations in the LTV-ratio both over time and across markets onto the same playing-field.

Our endogenous LTV-ratio is, in opposition to for instance Agarwal et al (2014), Allen and Carletti (2013) and Berger et al (2011), supply side driven. Focusing on mortgage supply, Borgersen and Robertsen (2012) shows that when regulations are insufficient, market developments, in particular expectations of continued collateral appreciation, might impact positively on the LTV-ratio as loss given default decreases. Goodhart and Hoffman (2008) argue that mortgagees might increase LTV-ratios to fulfill nominal return targets. Integrating both supply and demand side effects Lin (2014) applies a monetary general equilibrium model and shows how debtor welfare is not monotonically increasing in the LTV-ratio and that the optimal LTV-ratio both for the debtor and the creditor allows for the possibility of *ex post* default.

Analyzing supply side developments, financial innovations are important. Duca et al (2010) relates the rise in LTV ratios between 2000 and 2005 to two types of financial innovation originating on the supply side: credit scoring technology that enabled the sorting and pricing of non-prime mortgages and funding of such loans using collateralized debt obligations (CDOs) and credit default swaps (CDSs). There are also demand side links between LTV and risk.ⁱ Amior and Halket (2013) for instance, show how households average LTV-ratio has a strong negative correlation with house price volatility. Borgersen and Greibrokk (2012), analyzing mortgagor's the short term gains from different funding structures, allow leverage gains to stimulate the demand for higher LTV-ratios among mortgagors. A side effect of higher LTV-ratios is increased risk exposure for mortgagors.

Instead of focusing on either supply or demand side effects on the LTV-ratio Campbell and Hercowitz (2006) argue market innovations following the financial reforms of the early 1980s, in particular the Monetary Control Act of 1980 and the Garn-St.Germain Act of 1982, drastically reduced equity requirements associated with household borrowing.ⁱⁱ Arguing that changes in equity requirements follow regulatory changes, they treat equity requirements as

exogenous policy choices. Linking variations in the LTV-ratio to the regulatory and legal framework is accordance with Ahearne et al (2005). As both capital-adequacy and deposit guarantee schemes impact LTV-ratios in our model, we follow this line of reasoning, while acknowledging that market developments might take precedence when regulations are insufficient.

Relating risk pricing to the Furlong and Keeley (1987, 1989) framework we apply a parallel to the option-pricing approach in the sense that the mortgagor's equity stake and the LTV-ratio at origination is a key element. Incorporating the current LTV-ratio our approach is along the extensions of Ciochetti, Gao and Yao (2002) and others while, as we abstract away from mortgagors cash-flow position, the model is positioned within the equity theory of default.ⁱⁱⁱ The equity theory is for instance the starting point of Das and Meadows (2013) analyzing strategic default and highlighting the trade-off between future repayment and the probability of default that determines the optimal LTV-ratio.

Discussing risk based mortgage pricing Magri and Pico (2011) argue few papers to be concerned with risk based mortgage pricing while the ones that are centers around the US market. Bostic (2002) for instance argue that lenders, due to reduced storage costs for data and improved credit scoring techniques, started estimating the default risk of each borrower during the 90s. Recent US evidence is that easily collateralized household loans, such as mortgages, are those that have been most affected by these changes in pricing techniques (Magri and Pico, 2011, p. 1277).

The standard reference to the option-pricing literature on mortgages is Kau and Keenan (1995) while White (2004) provides an interesting supplement. Park and Bang (2014) argue the measurement of credit risk to involve three parameters; the possibility of default, loss-given defaults and the correlation across defaults, where there are few studies of loss-given default.^{iv} Distinguishing between risk pricing *ex ante* and *ex post* the paper explicitly account for the two former effects, where we separate the effect of a higher probability of default from that of increased costs in case of default. The model relates the latter to the change in the amount of collateralized debt that the distinction between the current LTV-ratio and the LTV-ratio at origination represents.

The Mortgagee

We consider a mortgagee that takes on deposits D , for which it pays a deposit rate r_D .

Deposits are conventionally insured by deposit insurance schemes (see for instance Kim, Kim and Han (2014)) The mortgagee's balance sheet identity states that mortgage supply L is constrained by mortgagee equity K and received deposits, $L = D + K$. The mortgagee uses housing as collateral for mortgages. The mortgagor (household) finances its purchase of a house with the value V by either equity E , or a mortgage L , giving the balance sheet identity $V = L + E$.

There is a probability p that the household may be able to pay back the mortgage and a probability $(1-p)$ that it may not. We refer to the case where the household is able to pay back a mortgage as success and the case where it may not as default.

In the absence of default is the mortgagee profit equal to expected operating income $p(r_u L - r_D D)$, where r_u is the mortgage rate. The mortgagee accepts a loan-to-value (LTV) ratio $\lambda = \frac{L}{V}$, which means that it - in case of default and with probability $(1-p)$ - covers its outstanding debt by its collateralized part of the housing value λV equating profit in case of default to $(1-p)(-L + \lambda V)$.

Expressing a mortgagee's expected profit as a function of the LTV-ratio $\pi = \pi(\lambda)$ we have

$$1) \quad \pi(\lambda) = p(r_u L - r_D D) + (1-p)(-L + \lambda V).$$

In the forthcoming sections we relate the LTV-ratio to the different components of the profit function to highlight their impact on the optimal LTV-ratio for a profit maximizing mortgagee.

Moral hazard

First, we introduce the probability of success as a decreasing function of the LTV-ratio, $p = p(\lambda)$, $p'(\lambda) < 0$. This automatically implies that the probability of default $(1-p(\lambda))$ is increasing in the LTV-ratio. The higher the LTV-ratio the lower (higher) is the probability of success (default). The existence of moral hazard in credit markets motivates this argument (see for instance Holmström and Tirole (1997) or Demirguc-Kunt and Detragiache (2002)). A higher LTV-ratio reduces (increases) the weight given to operating income (collateral) in the profit function.

The elasticity $\sigma_p = El_{p(\lambda)} = \left(\frac{\lambda}{p(\lambda)} \right) p'(\lambda)$ measures how the probability of success responds to a one percent increase in the LTV-ratio. The elasticity is an indicator of the extent of moral hazard in the mortgage market. From the sign of the derivative we know that the elasticity of success (or stated differently - the moral hazard elasticity) has a negative value $\sigma_p < 0$.

Risk pricing

Risk pricing is introduced by assuming a positive relation between the mortgage rate r_u and the LTV-ratio $r_u(\lambda)$ where $r_u'(\lambda) > 0$. (See for instance Kau and Keenan (1995) for the relation between the LTV-ratio and risk pricing). We assume deposits to be the only source of external funding, making the interest rate margin between the mortgage rate and the deposit rate. We simplify by assuming $r_D = 1$ and equate the mortgage rate $r_u(\lambda)$ to the interest rate margin. We operationalize the risk-pricing response $r_u'(\lambda)$ to the risk increase associated with a higher LTV-ratio by the risk pricing elasticity $\sigma_r = El_{r(\lambda)} = \left(\frac{\lambda}{r_u(\lambda)} \right) r_u'(\lambda)$.

Mortgage volumes

We allow for a positive relation between the mortgage volume and the LTV-ratio $L(\lambda)$ where $L'(\lambda) > 0$. The argument is that a higher LTV-ratio makes more households able to fulfill any given down-payment constraint and become mortgagors. As the number of mortgagors increase, so does aggregate lending. Gabriel and Rosenthal (2010) argues for a positive relation between the LTV-ratio and mortgage volumes. The lending elasticity $\sigma_L = El_{L(\lambda)} = \left(\frac{\lambda}{L(\lambda)} \right) L'(\lambda)$ measures mortgage response to a one percent increase in the LTV-ratio.

Collateral values

Finally, we allow for a positive impulse from LTV-ratios to house prices $V(\lambda)$ where $V'(\lambda) > 0$. The elasticity $\sigma_v = El_{V(\lambda)} = \left(\frac{\lambda}{V(\lambda)} \right) V'(\lambda)$ measures the extent to which house prices respond to a one percent increase in the LTV-ratio. The argument is part of the endogenous credit constraint of Kiyotaki and Moore (1997), where the LTV-ratio

represents the availability of credit. A positive relation between the LTV-ratio and house prices allows credit availability to impact positively on collateral values.^v

Two scenarios

We apply the model to consider two different scenarios derived from the relation between the initial LTV-ratio and the current LTV-ratio (defined as the mortgage volume relative to the market value).

When the mortgage volume relative to the market value of the house falls short of (or is equal to) the initial LTV-ratio $\lambda V \geq L \Leftrightarrow \lambda \geq \frac{L}{V}$, the expected loss given default is equal to what it was at origination, and is correspondingly priced into the current interest rate margin.

When, on the other hand, the mortgage volume (relative to the market value) exceeds the initial LTV-ratio $\lambda V < L \Leftrightarrow \lambda < \frac{L}{V}$ there is a potentially higher *default effect* on expected profits than what was the case at the time of origination. This risk increase is not priced in by the mortgagee and will impact risk pricing *ex post*.

The former scenario, where we in the following assume $\lambda = \frac{L}{V}$, describes the situation at origination where mortgage contracts are signed, and is in the fourth section used to derive the optimal LTV-ratio. The latter scenario describes a situation where depreciations have lifted the current LTV-ratio above the LTV-ratio at origination. This scenario is in the fifth section used to analyze a mortgagee's risk pricing response to a fall in house prices and a higher risk exposure for the mortgagee.

The profit maximizing LTV-ratio

We now consider a mortgagee at origination to find the profit maximizing LTV-ratio. When deriving the optimal LTV-ratio we remember how $\lambda = \frac{L}{V} \Rightarrow \lambda V = L$. At first, we abstract away from all but one of the presumed LTV effects described above, and then we include risk pricing, funding, lending and collateral as we go along.

Moral hazard: $p = p(\lambda)$

Expression 4) defines (implicitly) the optimal LTV-ratio λ^A as one that equates the response of risk pricing to that of moral hazard to a one percent increase in the LTV-ratio. The profit maximizing LTV-ratio is one where the mortgagee is paid for its actual risk exposure.

To give our argument some purchase we introduce welfare considerations by deriving the socially optimal LTV-ratio. We define the socially optimal LTV-ratio as one that maximizes a mortgagee's expected return $\max_{\lambda} p(\lambda)r_u(\lambda)L$. The first-order condition to this problem equals $-p'(\lambda)r_u(\lambda)L = p(\lambda)r'_u(\lambda)L$, which also is easily derived from expression 3).

The optimal LTV-ratio is, from a welfare point of view, equal to the LTV-ratio that comes about when lending is funded by mortgagee's own capital. Expression 4), and the LTV-ratio λ^A , may be seen as the socially optimal LTV-ratio.

In the following, we include different supply side features and see how these mortgage market characteristics create a wedge between this socially optimal LTV-ratio and the LTV-ratio maximizing mortgagee profits.

We start by bringing funding back into the game and allow for $D > 0$. The optimal LTV-ratio is again found by taking the derivative of 1) with respect to λ

$$5) \quad \frac{\delta\pi(\lambda)}{\delta\lambda} = p'(\lambda)(r_u(\lambda)L - D) + p(\lambda)r'_u(\lambda)L.$$

After some rearranging, we find the condition for $\frac{\delta\pi(\lambda)}{\delta\lambda} = 0$ as

$$6) \quad -p'(\lambda)(r_u(\lambda) - D/L) = p(\lambda)r'_u(\lambda).$$

The left hand side gives the marginal cost of a higher LTV-ratio and the right hand side the marginal gain. While the latter is related to the higher interest rate margin, is the former derived from the increased probability of default (and the reduced probability of success) that accompany a higher LTV-ratio.

Using the definition of σ_p we express the first-order condition as

$$7) \quad \lambda^B = \frac{\sigma_p(D/L - r_u(\lambda))}{r'_u(\lambda)},$$

where the optimal LTV-ratio is a function of the moral hazard elasticity, the interest rate margin, how tough risk pricing responds to higher risk and the capital adequacy ratio. (The inverse of the deposit to lending ratio, i.e. L/D , is in our model equal to the capital adequacy-ratio).^{vii} When operating income is positive, $r_u > D/L$, we see how the moral hazard elasticity, the capital-adequacy ratio and the degree of risk pricing impacts negatively on the LTV-ratio. The interest rate margin has, on the other hand, a positive impact on the LTV-ratio.

Alternatively, we may express the first-order condition in terms of elasticities, which is congruence with the expression in the absence of external funding and is the approach we pursue in the following

$$8) \quad -\sigma_p = \frac{\sigma_{r_u} r_u(\lambda)}{r_u(\lambda) - D/L}.$$

To highlight the funding effect we compare the optimal LTV-ratio λ^B derived from a situation with external funding to λ^A , the optimal LTV-ratio in the absence of external funding. The assumption of equal moral hazard technology^{viii} allows us to see directly from expressions 4) and 8) respectively, how

$$9) \quad \lambda^B = \lambda^A \frac{r_u(\lambda)}{(r_u(\lambda) - D/L)}.$$

Expression 9) shows a positive funding effect on the LTV-ratio $\lambda^B > \lambda^A$ as

$r_u(\lambda) > (r_u(\lambda) - D/L)$. When mortgagees fund lending by equity and deposits, lending may exceed mortgagee equity. This funding structure, where guarantee schemes insure deposits and a mortgagee does not carry all the risk associated with its lending activities, stimulates rather conventionally mortgagee risk taking and hence the optimal LTV-ratio.

Mortgage volumes: $p = p(\lambda), r_u(\lambda), D > 0$ and $L(\lambda)$

The second extension we pursue is a positive relation between mortgage volumes and the LTV-ratio. As mortgagees accept higher LTV-ratios, more households are able to fulfill the lower down-payment constraints, and become mortgagors, which again impact positively on aggregate lending.

When the lending effect is taken into account, a higher LTV-ratio has three effects on mortgagee profits: First, it reduces the probability of success. Second, it lifts the interest rate margin. Third, it increases mortgage volumes. Higher volumes introduce two additional effects on mortgagee profits; there is a positive impact on operating income and, as debt increases, the expected loss in case of default increases.

Taking the lending effect into account expression 1) equals

$$10) \quad \pi(\lambda) = p(\lambda)(r_u(\lambda)L(\lambda) - D) + (1 - p(\lambda))(-L(\lambda) + \lambda V),$$

and the corresponding first-order condition is

$$11) \quad \frac{\delta\pi(\lambda)}{\delta\lambda} = 0 \Leftrightarrow p'(\lambda)(r_u L - D) + p r_u'(\lambda)L + p r_u L'(\lambda) + (1 - p)(-L'(\lambda) + V) = 0.$$

After some rearranging (see the appendix for details) is the optimal LTV-ratio defined by

$$12) \quad -\sigma_p = \left(\frac{1}{r_u - D/L} \right) \left[r_u [\sigma_{r_u} + \sigma_L] + \frac{(1-p)}{p} (1 - \sigma_L) \right].$$

The left hand side is, as $\sigma_p < 0$, positive. The right-hand side has three components. The first two components represent *positive* effects on operating income from a higher LTV-ratio, due to a higher interest rate margin and increased lending, respectively. The latter effect, which comes from higher debt and increased losses in case of default, depends on the probability ratio and the value of the lending elasticity (a feature to be discussed more extensively later). The effect is negative when lending is elastic with respect to the LTV-ratio $\sigma_L > 1$ and positive when lending is inelastic $\sigma_L < 1$.

To find the impact of mortgage volumes on the LTV-ratio λ^C , which expression 12) defines, we compare expression 12) to expression 9) and see how

$$13) \quad \lambda^C = \lambda^B + \left(\frac{1}{r_u - D/L} \right) \left[r_u [\sigma_L] + \frac{(1-p)}{p} (1 - \sigma_L) \right].$$

From expression 13) we see that when $\left[r_u [\sigma_L] = \frac{(1-p)}{p} (\sigma_L - 1) \right]$ is there no lending effect on the LTV-ratio $\lambda^C = \lambda^B$. In this case is the expected increase in operating income equal to the expected increase in loss given default that accompany a higher LTV-ratio. When the interest

Thinking in terms of policy, we see easily which kind of measures that will be useful in constraining the LTV-ratio. Increased capital ratios counteract the funding effect, the lending effect and the collateral effect. When house prices are driven by fundamentals, and not the endogenous credit constraint (see for instance Borgersen (2016)), the collateral effect is missing. A credit risk policy dominated by debt-servicing ability and the first-line of defense ensures such a scenario (Borgersen, 2016). Finally, policies to reduce the lending effect are contingent on the moral hazard intensity. In the absence of moral hazard ($p \rightarrow 0$) a value on the lending elasticity that completely balances the effect of increased lending and higher operating income with the increase in debt and the effect on loss given default $\sigma_L = 1$ will curb the lending effect.^x When, on the other hand moral hazard is extreme ($p \rightarrow 1$) a mortgage policy that does not allow volumes to respond to higher LTV-ratios ($\sigma_L = 0$) will constrain the lending effect.

The risk pricing response to a fall in house prices

The last section showed how different mortgage market features impact the optimal LTV-ratio for a profit-maximizing mortgagee (at origination). In this section we apply the same framework to analyze the risk pricing response to a fall in house prices. Seeing the LTV-ratio as the risk exposure of a mortgagee, our framework allows for risk pricing assessments in relation to changes in the LTV-ratio. As we abstract away from the demand side of the mortgage market we assume house price changes to be sudden, implicitly taking away a mortgagor's option to increase repayments. This allows changes in house price to be passed-through to LTV-ratios, and the current LTV-ratio to differ from the LTV-ratio at origination.

We consider a case where, due to falling house prices, the current LTV-ratio exceeds the LTV-ratio at origination, $\lambda V < L \Leftrightarrow \lambda^{initial} < \frac{L}{V} \equiv \lambda^{new}$. A higher LTV-ratio increases a mortgagee's risk exposure compared to origination, and this additional risk is not priced in by the mortgagee. The question we address in this section is how risk pricing responds to a higher LTV-ratio. The mortgagee's optimization includes *ex post* what we refer to as a *default effect*.^{xi}

We start by introducing the derivative of expression 1) which has to take into account the fact that $\lambda V < L$. The first-order condition is

$$18) \quad \frac{\delta\pi(\lambda)}{\delta\lambda} = 0 \Leftrightarrow p'(\lambda)(r_u(\lambda)L - D) + p(\lambda)r_u'(\lambda)L - p'(\lambda)(-L + \lambda V) + (1 - p(\lambda))V = 0.$$

We consider a house price fall $\Delta V < 0$ which lifts the current LTV-ratio λ^{new} above the initial LTV-ratio $\lambda^{initial}$ where $\lambda^{new} = \lambda^{initial}(1 + a)$ and $a > 0$. When rearranged, the first-order condition can be expressed as

$$19) \quad r_u(\lambda) = D/L - \left(\frac{r_u'}{\sigma_p} \right) \lambda^{initial} + \left(\frac{(1-p)}{p} \right) \left(\frac{1}{\sigma_p} \right) \left(\frac{1}{1+a} \right) + \left(\frac{a}{1+a} \right)$$

The deposit to lending ratio gives the interest rate margin (or equivalently using our simplifications, the mortgage rate – which is the term we will apply in the following to highlight the risk pricing argument) necessary for zero net-operating profit. This first term of expression 19) is the break-even condition for a mortgagee. The second term relates the mortgage rate to a mortgagee's risk exposure at origination. This *ex ante* relation between the mortgage rate and the LTV-ratio $\lambda^{initial}$ is positive, as $\sigma_p < 0$. The latter two terms represent the *default effect*, measuring the additional risk exposure of a mortgagee that accompany a fall in house prices and a higher LTV-ratio. The increased risk exposure is both due to a higher probability of default and due to an increased cost in case of default.

The relation between the mortgage rate and the LTV-ratio is pictured in Figure 3, where the *ex post* relation differs from the *ex ante* relation. The prevailing risk exposure of a mortgagee is, when $a = 0$ and $\lambda^{initial} = \lambda^{new}$, assessed by the initial mortgage contract (*ex ante*). At origination the mortgagor has been offered an LTV-ratio $\lambda^{initial}$ for which she pays a mortgage rate $r_u^{initial}(\lambda) = D/L - \left(\frac{r_u'}{\sigma_p} \right) \lambda^{initial}$. The positively sloped line starting at the break-even condition (a mortgage rate equal to the D/L-ratio) shows that the higher the LTV-ratio is at origination, the higher is also the mortgage rate.

The *default effect* comes into play when $\lambda^{new} > \lambda^{initial}$ and $a > 0$.^{xii} As house prices have fallen and the risk exposure of a mortgagee has increased additional effects come into play in the risk assessments of a mortgagee. These effects create a kinked-relation between the mortgage rate and the mortgagee's risk exposure *ex post*.

First, is falling house prices impacting positively on the mortgage rate due to increased risk exposure among mortgagees $\lambda^{initial} < \lambda^{new}$, an effect we refer to as a *moral hazard effect*. The

LTV-ratio and the risk pricing response to a housing depreciation that increases ~~mortgagee~~ the risk exposure of a mortgagee.

First, the paper derives the optimal LTV-ratio for a mortgagee that supply mortgages using housing as collateral. Abstracting away from the demand side the paper highlights impacts on the LTV-ratio from the supply side of mortgage markets. The LTV-ratio measures the equity stake a mortgagor has in a house, and, naturally, lower equity increases the probability of default. Seeing the LTV-ratio as the risk exposure of a mortgagee variations in the LTV-ratio might be due to a number of factors, of which few are consistent with a view that the (inverse of-) LTV-ratio should be seen as an indicator of how mature a mortgage market is.

We benchmark our argument using a profit maximizing mortgagee funding mortgages using own equity. The profit maximizing LTV-ratio is one where mortgagees - at the margin - are paid for their actual risk exposure. Introducing welfare considerations, we find the socially optimal LTV-ratio to equal this LTV-ratio.

For a mortgagee that attracts external capital the optimal LTV-ratio exceeds the socially optimal LTV-ratio. When leveling the ~~playing field~~ playing field, we give the arguments regarding the relation between for instance deposit insurance and capital adequacy, moral hazard and risk pricing, lending volumes and collateral values and the profit maximizing LTV-ratio a common framework. Several of these arguments, and the variations in the LTV-ratio they create, do not stem from improved credit-scoring technology or more correct pricing of credit risk. Rather, they represent the opposite: Increased risk taking by mortgagees, or institutional arrangements protecting mortgagees from their risk exposure.

Highlighting supply side characteristics the paper shows how the supply side of mortgage markets might have been a key driver in the period preceding the financial crisis where LTV-ratios grew ~~were allowed to grow~~ to record highs. In the aftermath of the crisis macro-prudential interventions, both related to credit- and capital instruments, with the aim of amongst other things, constraining LTV-ratios, have come in place.^{xiii} Caps on LTV-ratios are basically interventions with the aim of constraining the funding effect, the lending and the collateral effect. While credit-related instruments such as LTV-caps are direct tools, are capital-related instruments are more indirect. The argument model shows however rather straightforward, ~~and in a non-technical manner~~ how capital adequacy rules can push the profit maximizing LTV-ratio closer to the ~~reduce the difference between the~~ socially optimal LTV-ratio. ~~and that which comes about when funding is external.~~ In fact, when considering

our optimal LTV-ratios (as given by expressions 13 and 17) the paper shows how stricter capital-adequacy ratios also reduce the lending and the collateral effect, pushing the LTV-ratio towards its socially optimal level. XXXX

The paper shows how external funding and problems related to asymmetric information and moral hazard, together with strategic behavior by mortgagees taking the lending effect and the collateral effect into account, might expose themselves to too high risk, allowing for higher LTV-ratios than what the supply side by itself, should support.

Second, the paper considers the risk pricing response to falling house prices and an LTV-ratio that exceeds its value at origination. We show how risk pricing *ex post ante* differs from that *ex antapost* as the *ex post* relation between the mortgage rate and the LTV-ratio is kinked. A shock to the LTV-ratio creates a moral hazard effect, a debt effect and a regime effect on risk pricing. The ~~two~~-latter two creates together a default ~~effect which~~effect that impacts positively on loss given default and increases the risk pricing response to a higher LTV-ratio *ex post*. This default effect is initially not priced in by the mortgagee and produces a kinked relation between the LTV-ratio and the mortgage rate *ex post* -which is in accordance with risk pricing terminology.

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Appendix 1

This appendix derives the optimal LTV-ratio for a mortgagee that takes on deposits and supply mortgages using housing as collateral. First, we find the optimal LTV-ratio when abstracting away from both the lending and the collateral effect. We start out by considering the profit function

$$A1) \quad \pi(\lambda) = p(\lambda)(r_u(\lambda)L - r_D D) + (1 - p(\lambda))(-L + \lambda V),$$

and take the derivative with respect to λ (As $\lambda V = L$ the last term disappears.)

$$A2) \quad \frac{\delta\pi(\lambda)}{\delta\lambda} = p'(\lambda)(r_u(\lambda)L - D) + p(\lambda)r_u'(\lambda)L.$$

The condition for $\frac{\delta\pi(\lambda)}{\delta\lambda} = 0$ is

$$\text{A3)} \quad -p'(\lambda)\left(r_u(\lambda) - \frac{D}{L}\right) = p(\lambda)r_u'(\lambda).$$

Using the definition of the moral hazard elasticity σ_p and the definition of the risk pricing elasticity σ_{r_u} the first-order condition equals

$$\text{A3')} \quad -\sigma_p = \frac{\sigma_{r_u} r_u}{r_u - D/L}.$$

In the absence of external funding ($D=0$) this reduces to

$$\text{A4)} \quad -\sigma_p = \sigma_{r_u}.$$

For a profit maximizing mortgagee the optimal LTV-ratio is characterized by equality between the moral hazard elasticity and the risk pricing elasticity, and is correspondingly one where mortgagees are paid for their risk exposure at the margin.

While A3') equals expression 8), is expression A4) equal to expression 3). As expression 3)

defines λ^A and expression 8) defines λ^B we find $\lambda^B = \lambda^A \frac{r_u}{\left(r_u - \frac{D}{L}\right)}$.

The lending effect

Taking the lending effect $L(\lambda)$ into account, expression 1) is written as

$$\text{A5)} \quad \pi(\lambda) = p(\lambda)(r_u(\lambda)L(\lambda) - r_d(\lambda)D) + (1-p(\lambda))(-L(\lambda) + \lambda V).$$

Taking the derivative with respect to the LTV-ratio (remembering that $\lambda V = L$) gives

$$\text{A6)} \quad \frac{\delta\pi(\lambda)}{\delta\lambda} = p'(\lambda)(r_u L - D) + p r_u'(\lambda)L + p r_u L'(\lambda) - p'(\lambda)(-L + \lambda V) + (1-p)(L'(\lambda) + V),$$

where the condition for $\frac{\delta\pi(\lambda)}{\delta\lambda} = 0$ equals

A7)

$$L p'(\lambda)\left(r_u - \frac{D}{L}\right) + L p\left[r_u'(\lambda) + r_u \frac{L'(\lambda)}{L}\right] - p'(\lambda)L(-1 + \lambda V/L) + (1-p)L\left(\frac{V}{L} - \frac{L'(\lambda)}{L}\right) = 0.$$

Deleting L , and once again using $\lambda = \frac{L}{V}$, we have

$$\text{A8)} \quad p'(\lambda)\left(r_u - \frac{D}{L}\right) + p\left[r_u'(\lambda) + r_u \frac{L'(\lambda)}{L}\right] + (1-p)\left(\frac{1}{\lambda} - \frac{L'(\lambda)}{L}\right) = 0.$$

This can be rewritten as

$$A9) \quad p'(\lambda)(r_u - D/L) + p \frac{1}{\lambda} \left[\lambda r_u'(\lambda) + r_u \frac{\lambda L'(\lambda)}{L} \right] + (1-p) \frac{1}{\lambda} \left(1 - \frac{\lambda L'(\lambda)}{L} \right) = 0.$$

When applying the definition of the lending elasticity σ_L , and multiplying by λ , A9) equals

$$A10) \quad \lambda p'(\lambda)(r_u - D/L) + p \left[\lambda r_u'(\lambda) + r_u \sigma_L \right] + (1-p)(1 - \sigma_L) = 0.$$

Dividing by p and using the definition of the moral hazard elasticity gives

$$A11) \quad \sigma_p (r_u - D/L) + \left[\lambda r_u'(\lambda) + r_u \sigma_L \right] + \frac{(1-p)}{p} (1 - \sigma_L) = 0.$$

Finally, and now we only consider the middle-term, we place the interest rate margin outside the parenthesis and apply the definition of the risk pricing elasticity σ_p , to have

$$A12) \quad \sigma_p (r_u - D/L) + r_u \left[\sigma_{r_u} + \sigma_L \right] + \frac{(1-p)}{p} (1 - \sigma_L) = 0.$$

Rearranged in terms of the moral hazard elasticity (and remember how $\sigma_p < 0$) this equals

$$A13) \quad -\sigma_p = \left(\frac{1}{r_u - D/L} \right) \left[r_u \left[\sigma_{r_u} + \sigma_L \right] + \frac{(1-p)}{p} (1 - \sigma_L) \right].$$

A13) - which defines λ^C - is equal to expression 12). Comparing A13) to A3), knowing that the moral hazard technology is equal across our regimes, allows us to see how expression 13) comes about as

$$A14) \quad \lambda^C = \lambda^B + \left(\frac{1}{r_u - D/L} \right) \left[r_u \left[\sigma_L \right] + \frac{(1-p)}{p} (1 - \sigma_L) \right].$$

The collateral effect

Taking the collateral effect $V(\lambda)$ into account, expression 1) equals

$$A15) \quad \pi(\lambda) = p(\lambda)(r_u(\lambda)L(\lambda) - r_D(\lambda)D) + (1-p(\lambda))(-L(\lambda) + \lambda V(\lambda)).$$

The first-order condition is found by considering the derivative

$$A16) \quad \frac{\delta \pi(\lambda)}{\delta \lambda} = p'(\lambda)(r_u L - D) + p r_u'(\lambda)L + p r_u L'(\lambda) - p'(\lambda)(-L + \lambda V) + (1-p)(L'(\lambda) + V + \lambda V'(\lambda))$$

Now, using the exact same steps as above - from A8) to A14) - with the only extension being that the definition of the collateral elasticity σ_L is included in the last term, expression A17)

defines the optimal LTV-ratio in the presence of collateral λ^D (as given by expression 16) as

$$A17) \quad -\sigma_p = \left(\frac{1}{r_u - D/L} \right) \left[r_u [\sigma_{r_u} + \sigma_L] + \frac{(1-p)}{p} (1 - \sigma_L + \sigma_V) \right].$$

When compared to expression A14), expression A17) reproduces expression 17) as

$$A18) \quad \lambda^D = \lambda^C + \left(\frac{1}{r_u - D/L} \right) \left[\left(\frac{1-p}{p} \right) \sigma_V \right].$$

Appendix 2

Appendix 2 derives the risk pricing response to a fall in house prices. Here we apply an extended version of expression 1) where the current LTV-ratio differs from the LTV-ratio at origination $\lambda V < L \Leftrightarrow \lambda^{initial} < \frac{L}{V} \equiv \lambda^{new}$. We fix $\sigma_L = \sigma_V = 0$, as neither lending volumes nor collateral values can be argued to respond positively to a fall in house prices *ex post*.

The derivative of expression 1) now equals

$$A19) \quad \frac{\delta\pi(\lambda)}{\delta\lambda} = p'(\lambda)(r_u(\lambda)L - D) + p(\lambda)r_u'(\lambda)L - p'(\lambda)(-L + \lambda V) + (1 - p(\lambda))V$$

We now consider a drop in house prices $\Delta V < 0$ that lifts the current LTV-ratio above the LTV-ratio at origination $\lambda^{new} > \lambda^{initial}$ and assume $\lambda^{new} = \lambda^{initial}(1+a)$ where $a > 0$. The first-order condition $\frac{\delta\pi(\lambda)}{\delta\lambda} = 0$ can then be rewritten as

$$A20) \quad p'(\lambda)(r_u(\lambda) - D/L) + p(\lambda)r_u'(\lambda) - p'(\lambda)(-1 + \lambda(V/L)) + (1 - p(\lambda))V/L = 0.$$

Inserting for $\lambda^{new} = L/V$ and $\lambda^{initial}$ respectively, gives

$$A21) \quad p'(\cdot)(r_u(\cdot) - D/L) + p(\cdot)r_u'(\cdot) - p'(\cdot)(-1 + \lambda^{initial}/\lambda^{new}) + (1 - p(\cdot))1/\lambda^{new} = 0$$

When dividing by $p'(\cdot)$, inserting for, $\lambda^{new} = \lambda^{initial}(1+a)$ using the definition of the moral hazard elasticity σ_p and solving for the mortgage rate (or equivalently the interest rate margin) we have

$$A22) \quad r_u(\lambda) = D/L - \left(\frac{r_u'}{\sigma_p} \right) \lambda^{initial} + \left(\frac{a}{1+a} \right) + \left(\frac{(1-p)}{p} \right) \left(\frac{1}{\sigma_p} \right) \left(\frac{1}{1+a} \right)$$

The two latter terms are conditional on $a > 0$ and $\lambda^{new} > \lambda^{initial}$ and represent the *default effect*. As house prices fall and the risk exposure of a mortgagee increases, the new (and higher) LTV-ratio introduces two additional effects on the relation between the mortgage rate and the LTV-ratio. One of these effects $\left(\frac{1-p}{p}\right)\left(\frac{1}{\sigma_p}\right)\left(\frac{1}{1+a}\right)$ is due to the higher probability of default (*the regime effect*), while the other $\left(\frac{a}{1+a}\right)$ comes from a mortgagee's higher amount of non-collateralized debt (*the debt effect*). Together these two effects represent the increase in loss given default that comes about when house prices fall and the risk exposure of a mortgagee increases, and is referred to as the *default effect* in the model.

Appendix 3

Appendix 3 derives the optimal LTV-ratio in the case of an integrated lending- and collateral effect. We start out from a relation between the LTV-ratio, the mortgage volume and the collateral value given by $L = L(V(\lambda))$ where $L'(V) > 0$ and $V'(\lambda) > 0$. The derivative of the lending volume with respect to the LTV-ratio equals

$$A23) \quad \frac{\delta L}{\delta \lambda} = \frac{\delta L}{\delta V} \frac{\delta V}{\delta \lambda} = \frac{\delta L}{\delta V} \frac{\delta V}{\delta \lambda} \frac{L}{L} \frac{V}{V} \frac{\lambda}{\lambda} = \sigma_{LV} \sigma_V V.$$

The second equality sign comes about by multiplying both the nominator and the denominator by L , V and λ , while the last equality sign comes from using the definition of the collateral elasticity $\sigma_V = El_{V(\lambda)} = \left(\frac{\lambda}{V(\lambda)}\right)V'(\lambda)$ as presented earlier, and introducing the lending to collateral elasticity $\sigma_{LV} = El_{L(V)} = \left(\frac{L}{V}\right)L'(V)$.

Using expression 14), which we reproduce as A24), taking the integrated lending effect into account,

$$A24) \quad \pi(\lambda) = p(\lambda)(r_u(\lambda)L(V(\lambda)) - r_D(\lambda)D) + (1-p(\lambda))(-L(V(\lambda)) + \lambda V(\lambda)),$$

the first-order condition equals

$$A25)$$

$$\frac{\delta \pi(\lambda)}{\delta \lambda} = 0 \Leftrightarrow p'(\lambda)(r_u L - D) + p r_u'(\lambda)L + p r_u L'(V)V'(\lambda) + (1-p)(-L'(V)V'(\lambda) + V + \lambda V'(\lambda)) = 0$$

Rearranging, and expressing the first-order condition in terms of elasticities, the optimal LTV-ratio (for an integrated lending- and collateral effect) λ^{lte} is defined by

$$A26) \quad -\sigma_p = \left(\frac{1}{r_u - D/L} \right) \left[r_u [\sigma_{r_u} + \sigma_{LV} \sigma_V] + \frac{(1-p)}{p} (1 + \sigma_V (1 - \sigma_{LV})) \right].$$

When compared to A3'), that is the step before introducing lending and collateral in the baseline model, but allowing for external funding, the optimal LTV-ratio may be expressed as

$$A27) \quad \lambda^{Inte} = \lambda^B + \left(\frac{1}{r_u - D/L} \right) \left[r_u \sigma_{LV} \sigma_V + \left(\frac{1-p}{p} \right) (1 + \sigma_V (1 - \sigma_{LV})) \right].$$

When compared to A18) we see how the optimal LTV-ratio is higher with an integrated lending-collateral effect relative to when the two effects are separated as long as the interest rate margin (the mortgage rate) exceeds a critical level.

$$A28) \quad \left[r_u > \frac{\left(\frac{1-p}{p} \right) \sigma_{LV} (\sigma_V - 1) - 1}{\sigma_{LV} \sigma_V} \right].$$

While more complex than A18), where the lending effect is separated from the collateral effect, and remembering the different definitions of the lending elasticity and the lending to collateral elasticity, it is still a critical mortgage rate that ensures a higher LTV-ratio.

ⁱ See also Campbell and Cocco (2003) or Cocco (2004) for the role of housing in household's portfolio risk and Hryso et al (2010) for house prices and the risk exposure of households.

ⁱⁱ See Borgersen (2015b) - and the references therein - for the relation between mortgage demand and the LTV-ratio.

ⁱⁱⁱ The seminal paper by Jackson and Kasserian (1980) distinguishes between two basic views on mortgage default, the equity theory of default and the ability-to pay theory of default, respectively.

^{iv} See the references in Park and Bang (2014) for the loss severity of mortgagee exposure.

^v This framework separates the lending- from the collateral effect, in order to analyze the two effects separately. The alternative would be to integrate the two, making the mortgage volume a function of collateral, which again is related to the LTV-ratio $L(V(\lambda))$ where $L'(V) > 0$ and $V'(\lambda) > 0$. Appendix 3 derives the optimal LTV-ratio for an integrated lending-collateral effect where results are qualitatively equal to those of the main model.

^{vi} While our argument is related to moral hazard, differences in for instance household income, and hence debt-servicing ability could alternatively be applied to argue for different LTV-ratios. For a discussion of mortgagor characteristics and mortgagee risk see, for instance, Diaz-Serrano (2004) or Leece (2004).

^{vii} Our model gives a conventional role to both external funding in general and capital-adequacy more specific. This allows for a straightforward argumentation regarding the impact of external funding on the LTV-ratio. While deposit insurance impacts positively, is the impact from tighter capital adequacy rules on the LTV-ratio negative. See for instance Anginer et al (2014) for a general discussion on deposits guarantee schemes and Van Hoose (2007) for a survey of the (theoretical) literature on capital ratios.

^{viii} The moral hazard technology is represented by the probability function $p = p(\lambda)$, $p'(\lambda) < 0$ and equal moral hazard technology equates the probability function, which implies that the elasticity of success (the moral hazard elasticity) $\sigma_p = \left(\frac{\lambda}{p(\lambda)} \right) p'(\lambda)$ only differs with respect to the LTV-ratio itself, when comparing scenarios A and B.

^{ix} Alternatively, we may express the condition for a positive lending effect in terms of a maximum value for the lending elasticity $\frac{1}{1-p r_u / (1-p)} > \sigma_L$. When $p=0$ inelastic lending, $\sigma_L < 1$, is the condition for a positive lending

effect. When $p > 0$ is a positive lending effect compatible with elastic lending $\sigma_L > 1$.

^x The lending effect is determined by the parenthesis in expression 13), which may be expressed as $[p r_u [\sigma_L] + (1-p)(1-\sigma_L)]$. When $(p \rightarrow 0)$ we see that $\sigma_L = 1$ eliminates the lending effect while $\sigma_L = 0$ does the same when $(p \rightarrow 1)$.

^{xi} When shifting from an *ex ante* to an *ex post* scenario, we keep $\sigma_L = \sigma_V = 0$ as neither lending volumes nor collateral values realistically would respond positively to a fall in house prices *ex post*. See Appendix 2 for details.

^{xii} The reasoning of the model implies symmetric effects in the sense that $\lambda^{initial} > \lambda^{new}$ and a < 0 creates a situation with a lower probability of default and a reduced cost of default, which, analogue to the case of depreciations, will have implications for risk pricing during periods of appreciations.

^{xiii} See for instance Gelati and Moessner (2011) or Lim et al (2011) for a review of the literature on Macprudential Policy and Taylor (2009) for a discussion of the policy response(s) to the financial crisis.