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Teachers' framing of students' difficulties in mathematics learning in collegial discussions

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ABSTRACT

This study investigates the diagnostic and prognostic framings of Swedish mathematics teachers regarding the difficulties experienced by students in mathematics learning. Collegial discussions among 65 mathematics teachers in nine collegial groups were videotaped during a professional development (PD) program entitled Boost for Mathematics for analysis. The results show that the diagnostic framings of the teachers were mainly attributed to the cognitive abilities of students, whereas the prognostic framings were mainly related to lesson organization such that students should collaborate. While the teachers emphasize collaborate group work, they put little emphasis on how they could act in these learning situations. These results contribute to the understanding of Swedish mathematics teachers' framing of students' difficulties in mathematics learning and to the role of collegial discussions in PD initiatives.

KEYWORDS

Student difficulties; mathematics teachers; diagnostic framing; prognostic framing; collegial discussion

Introduction

Reforms in mathematics education have involved the reorganization and development of syllabi, curriculum materials, and classroom practices. However, professional development (PD), which enables teachers to establish further productive classroom practices, is central to these policy changes. A key part of PD programs is collegial discussions among teachers (Cobb et al., 2018; Cobb & Jackson, 2011; Munter & Wilhelm, 2021). However, the opportunities for teacher development are dependent on the characteristics of these collegial discussions (Desimone, 2009; Munter & Wilhelm, 2021). Scholars suggested that instructional reform is closely related to the views of teachers regarding the mathematical capabilities of students (Jackson et al., 2017; Jackson et al., 2018) as well as the provision of learning opportunities for students (Jackson, 2011; Russo et al., 2020). For example, Horn (2007) found that this view affected teacher's classroom actions, whereas Boaler (2015) and Sun (2019) classified teachers as having either a fixed mindset or growth mindset. With a fixed mindset, teachers tend to exclude students with difficulties from rigorous mathematical activities. With a growth mindset, however, teachers encourage students to contribute and to be successful. Various studies classified teacher differences based on their framing of the mathematical capabilities of the students and their reactions to such capabilities (Anthony et al., 2018; Bannister, 2015; Jackson et al., 2017; Wilhelm et al., 2017).

With a specific focus on diagnostic and prognostic teacher framings, the current study analyzed collegial teacher discussions to examine their perception of the mathematical capabilities of

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students. Specifically, mathematical capabilities are related to an aspect of reform-oriented instruction, that is, the view that students are capable of participating in rigorous mathematical activities such as collaborative learning centered around cognitively demanding mathematical tasks (cf. Franke et al., 2007; Lahann & Lambdin, 2020). The framing problems of practice are inherently a social process (Bannister, 2015; Jackson et al., 2017). When approaching teachers' framing of students' capabilities in mathematics learning, we focus on the framing of a common problem in practice (Jackson et al., 2017), which is students facing difficulties in mathematics learning. The conceptualization of teachers' framing of students' difficulties in mathematics learning is connected to difficulties in participating in rigorous mathematical activities, which involve mathematical abilities (cognitive skills) and other types of difficulties experienced by students, such as attention and motivation. Thus, this study poses the following research question: "How do teachers explain the source of their students' difficulties in mathematics learning, and how do they support these difficulties?" The study proposes that teachers' views regarding students' difficulties in mathematics learning are important when intending to accomplish instructional reform (Jackson et al., 2017). "Boost for Mathematics" (Skolverket, 2018) is an example of such a reform initiative for assisting teachers in establishing productive classroom practices. Therefore, the study posits that examining teachers in their collegial discussion will contribute to the understanding of teacher engagement in reform efforts.

On the basis of earlier studies (e.g., Ryve & Hemmi, 2019), we conjecture that the prevalent ideas of instructional practices in Sweden heavily influenced the majority of the teachers' perceptions of students and their instructional practices (e.g., Ryve et al., 2016) instead of productive practices advocated within the Swedish mathematics reform movement. Specifically, we predicted that teachers would adopt aspects of the reform movement that were in agreement with their dominant visions of instructional practices and would dismiss reform initiatives that do not.

Research background

To add to the understanding of the impact of the reform movement in mathematics education, this study examined the reasoning of teachers regarding the difficulties experienced by students in mathematics learning through the national PD entitled, "Boost for Mathematics." This initiative for mathematics is a national state-coordinated project initiated by the government and includes a year-long PD program offered to all mathematics teachers in Sweden during the period from 2012 to 2016 (cf. Lindvall et al., 2018). The central tenet of this education reform movement in mathematics is to assist teachers in establishing productive learning practices in mathematics, which have been found challenging and difficult (Desimone & Garet, 2015; Garet et al., 2011). The vision of instruction required by the policy documents and the abovementioned Boost for Mathematics represent a radical departure from the traditional teaching practice for mathematics. This vision involves new perspectives on teaching and learning, such as eliciting and building upon student thinking (Munter & Wilhelm, 2021). A central part of this vision of instruction is viewing students as capable of participating in rigorous mathematical activities (Munter, 2014). As the teachers' perceptions of the mathematical capabilities of students are particularly important when implementing ambitious instructional reforms (Jackson et al., 2017), focusing on these perceptions in PD environments is vital to understanding the success of such reforms.

In Sweden, mathematics standards at the national level, which are mainly influenced by the National Council of Teachers of Mathematics Standards (cf. Boesen et al., 2014), were implemented in 1994 and 2011. The motives behind the reforms were to deviate from the traditional teaching of mathematics, which mainly focuses on procedural knowledge (Rittle-Johnson et al., 2001), and to emphasize broad competence goals, such as problem-solving or reasoning skills. Recently, Forsberg et al. (2016) identified another common practice in Sweden, which is a democratic teacher practice, that is present and that was relatively predominant during the 1990s and 2000s. Its key components are equal opportunities and equal rights. In addition, it aims to foster democratic citizenship and the development of a democratic mindset and skills (Forsberg et al., 2016). Such teaching practices are reflected in the Swedish curriculum (Skolverket, 2011), which includes, for example, showing respect, taking responsibility, and developing a critical and reflective attitude.

Diagnostic and prognostic framings

To understand the teachers' perceptions of students' capabilities, many studies (Anthony et al., 2018; Bannister, 2015; Jackson et al., 2017) have used problem framing (Goffman, 1974; e.g., diagnostic framing, which involves the identification of a problem and attribution of blame) and prognostic framing (involves proposing a solution to a diagnosed problem that specifies the actions needed). For example, Jackson et al. (2017) aimed to assess the effect of instructional reforms and examined the views of middle-grade teachers regarding the mathematical capabilities of students using a qualitative semi-structured interview. They applied productive and unproductive diagnostic and prognostic framings to elucidate how teachers signaled encouragement or discouragement toward students with difficulties. The authors then classified these opinions into four dimensions. The first is *productive diagnostic framing*, in which students' difficulties in mathematics learning were attributed to instructional or schooling opportunities. Next, students' difficulties in mathematics learning were attributed to inherent student traits from the perspective of *unproductive diagnostic framing* (e.g., processing speed [whether students are fast or slow] and motivation [whether students are lazy] (Bailey et al., 2014), and that several students are naturally better at mathematics than others are) or family or community deficits. Third, *productive prognostic framing* enables students with difficulties to participate in rigorous mathematical activities. Lastly, *unproductive prognostic framing* reduces the rigor of learning goals for students with difficulties. Jackson et al. (2017) concluded that the majority of teachers held the unproductive diagnostic view. In other words, they did not view all students as capable of participating in rigorous mathematical activities and believed that such difficulties were inherent traits or the result of family or community deficits. Moreover, these teachers displayed an unproductive prognostic framing when they suggested that the cognitive demand of activities should be lessened when students are experiencing difficulties.

Moreover, Horn (2007) compared the conversations of two groups of mathematics teachers in high school when discussing the "mismatch problem," that is, the mismatch between the teachers' framing of their students' abilities and an equity-geared reform curriculum. The productive and unproductive framing analyses of the teachers' meeting identified how they made sense of students with difficulties. Several teachers viewed these difficulties as related to inherent student traits, whereas others perceived these difficulties as related to learning opportunities within the classroom. One group of teachers blamed the students and claimed that the students were not engaged or did not work hard enough, whereas another group placed more responsibility on the teachers to address potential mismatches between the curriculum and students by considering the means through which teachers could best alter their instruction to support the learning of students.

Building on Horn (2007), Wilhelm et al. (2017) investigated the extent to which teachers' explanations regarding the sources of the difficulties of students in mathematics were related to students' participation in quality mathematics discourse. The investigation, which involved video recordings of classroom instruction, teacher interviews, and written assignments, was conducted over four years with 156 mathematics teachers in middle school. The annual interviews, which were focused on the diagnostic framings of the teachers regarding the mathematics difficulties of students, were then categorized as productive, unproductive, or mixed, that is, wavering between a productive and an unproductive explanation, and were linked to the distribution and quality of the students' mathematical discourse. The contributions and participation of the students in classroom discourse were measured using three indices, namely, providing an account of their reasoning, linking to and building on one's ideas, and the percentage of students who participated in class discussion. The authors deduced that the majority of teachers believed that the mathematics difficulties of students were related to their participation in mathematical discourses in the classroom. As a result, the students were more likely to participate in discussions if the teacher diagnosed the sources of such difficulties in mathematics as related to the nature of instruction or learning opportunities. Moreover, teachers believed that the mathematics capabilities of students were related to the quality of learning opportunities given to them by teachers.

Challenging the teachers' views of students' capabilities

When teachers first engage in professional discussions regarding students' mathematical capabilities, they typically focus on what students are unable to do (Kazemi & Franke, 2004) and make definitive, evaluative claims regarding students level of knowledge (Gamoran Sherin & Van Es, 2009). When explaining mathematical capabilities, many teachers refer to students as being *low* or *high* level or functioning below or above a grade level (Boaler, 2015; Sun, 2019). Moreover, many teachers believe that mathematical ability is an innate gift that people may or may not possess. These beliefs regarding students' ability are in agreement with Dweck's (2000) mindset theory, which was further elaborated by Boaler (2015). A growth mindset is a belief that everybody has the ability to improve and that student abilities can be developed through instruction. In other words, it focuses on what students are capable of doing, where teachers then leverage the existing understanding and experience to design an instruction that matches learner needs.

PD which focuses on student thinking and the lens of problem framing to provide insight into teachers' views of their students' capabilities can result in changes in the teachers' discourses regarding the students as learners (Horn, 2007). Wilson et al. (2017) examined discussions among 22 teachers in elementary school participating in a year-long 60-hour mathematics PD program. The authors identified four categories related to the mathematics learning performances of students, namely, age or grade level (age/grade); fixed, innate ability (ability); effort of a student (effort); and the degree to which the student was fortunate to obtain the correct answer (luck). However, the evidence of the terminology and ideas from the PD program were found limited when the teachers discussed the effort- and luck-based explanations. However, changes were observed when they discussed age/grade and ability as the teachers incorporated the PD ideas over time. Near the end of the program, the teachers used productive age/grade and ability framings to explain the mathematical activities of the students. Anthony et al. (2018) examined teachers within a whole-school PD program to understand if the program led to any changes in the views of the teachers regarding the capabilities of students. The study concluded that nearly all teachers displayed positive shifts toward productive diagnostic and prognostic framings over time. However, one teacher, who continued to assess students in the unproductive diagnostic category, remained concerned that struggling students lacked sufficient knowledge and continued to categorize them on the basis of the presence or absence of knowledge.

Thus, a general consensus exists for a set of goals for the mathematical learning of students and is represented in the Swedish national curriculum (Boesen et al., 2014), which is influenced by the Principles and Standards for School Mathematics (NCTM, 2000) and Adding it Up (Kilpatrick et al., 2001). These learning goals emphasize the development of procedural fluency, conceptual understanding of mathematical reasoning, and ability to communicate mathematical ideas, which are a part of the mathematical proficiency for students (Kilpatrick et al., 2001). Teachers engaged with education reform movements for mathematics, such as Boost for Mathematics, will need to address the key aspects of teaching that support the students' attainment of such learning goals. Previous studies focused on whether teachers hold a productive or nonproductive view of students' mathematical capabilities (Anthony et al., 2018; Jackson et al., 2017; Wilson et al., 2017). Given the complex learning goals for mathematical learning, investigating whether teachers differently frame the difficulties of students in mathematics learning on the basis of the abovementioned learning goals will be essential. Therefore, the view of teachers regarding students as capable of participating in such classroom activities is important for understanding how teachers engage in such activities. From this research background there is a need to know more about how teachers in their collegial discussion frame their students' difficulties. Therefore, our research questions is, "How do teachers explain the source of their students' difficulties in mathematics learning, and how do they support these difficulties?" Further, Jackson et al. (2017) found that the diagnostic and prognostic framing dimensions provided useful guidance when assessing the conversations of teachers with colleagues regarding the mathematical capabilities of students. Therefore, inspired by Jackson et al. (2017), this study examined collegial teacher discussions regarding the mathematical difficulties of students during a PD program in Sweden using the diagnostic and prognostic framing dimensions.

Methodology

This study aimed to understand how teachers characterized the difficulties of their students in mathematics learning with a specific focus on the framings (i.e., diagnostic or prognostic) used to describe the sources of such difficulties. Grounded in Goodwin's (1994) belief that discourse is constitutive of a practitioners' professional vision, the understanding of the discourse was focused on how teachers made sense of the phenomena in their work domain.

Study context

Between 2013 and 2016, the Swedish National Agency for Education launched a curriculum-based PD project called "Boost for Mathematics" worth 649 million kroner (approximately 65 million euro; Skolverket, 2018) to strengthen the quality of mathematics teaching and student performance. The central components of this project were 24 modules, 8 per grade level range (1–3, 4–6, and 7–9), which were developed to support teaching teams to plan, establish, and reflect on pedagogical mathematics practices. Each module was designed to assist teaching groups (in one semester) in engaging in eight iterations. Each module included individual preparation (session A), collegial discussions on resources and lesson planning (session B), individual classroom teaching based on content (session C), and collective reflections on classroom instruction and process (session D). Teachers were required to complete two modules of the PD program by taking one module per semester. The PD sessions were held at each school with the support of a trained coach. In the research project, which is funded by the Swedish National Research Council, a large data set was constructed during the academic year 2015–2016. Mathematics teachers and coaches from several large municipalities in Sweden teaching grades 1–9 were invited to participate in the data collection process if they were about to start the Boost for Mathematics program in the autumn of 2015. For detailed descriptions of the PD program, see Boesen et al. (2015) and Lindvall et al. (2018).

Data collection

The research presented in this paper is a part of a research project[1] that involves a large data set constructed from 2015 to 2016, which consists of video-recorded mathematics lessons, collegial meetings, and interviews with more than 200 teachers from 17 schools in three municipalities who participated in the PD program "Boost for Mathematics." The study retrieved data from the large data set, which consisted of video recordings of collegial discussions among nine teacher groups with a total of 65 teachers under the "Boost for Mathematics" PD project. Table 1 provides an overview of the participants and data collection. All teachers provided ethical consent to use the data in this research. The nine groups were selected using convenience sampling with the objective of covering all thematic modules from "Boost for Mathematics" and covered all grade levels (i.e., 1–3, 4–6, and 7–9). The data set from the schools covered six of the eight thematic modules. The only missing modules were geometry and probability and statistics because none of the 17 large teacher groups in the data set opted to study these subjects.

Table 1. Overview of the participants and data collection.

Group	Grade levels	Number of participants	Module	Video-recorded meetings
1	1–3	6	Problem solving	Session B: 50 min Session D: 49 min
2	4–6	8	Algebra	Session B: 88 min Session D: 70 min
3	1–3	8	Algebra	Session B: 57 min Session D: 49 min
4	4–6	6	Number sense and use of numbers	Session B: 67 min Session D: 64 min
5	1–3	9	Language in mathematics	Session B: 62 min Session D: 42 min
6	7–9	7	Algebra	Session B: 46 min Session D: 37 min
7	4–6	8	Language in mathematics	Session B: 55 min Session D: 48 min
8	1–3	7	Correlations and change	Session B: 64 min Session D: 52 min
9	7–9	6	Mathematics instruction with ICT	Session B: 68 min Session D: 53 min

The number of members per group, including the coach, ranged from four to nine. Therefore, 65 teachers participated in the research, with their years of experience ranging from 1 to more than 30 years. The primary data were videotapes from two meetings of each group. The first was on collective planning with colleagues (session B), and the second was on collective classroom instruction reflections (session D). Both meetings were conducted around a table at the teachers' workplaces and were documented using two video cameras operated by assistant researchers and an audio recorder placed in the middle of the table. The document data were later combined into one video/audio file. The meetings were video-recorded to enable data sharing and to examine specific time segments from various analytical foci.

Analysis

Building on Bannister's (2015) perspective, a framing lens was used to assess the teachers' reasoning regarding students' difficulties in mathematics learning. The frames were viewed as coconstructed objects that represent certain meanings in the groups of teachers at particular times in their collegial discussions on various topics, such as teacher practices, student perspectives, classroom organization, and lesson planning. Therefore, in the first phase of analysis, meaningful utterances in each group meeting were identified and defined as a unit of teacher discourse on students' capabilities. Each episode that identified pedagogical reasoning (Horn, 2007), such as descriptions of student issues accompanied by detailed reasons, explanations, or justifications, was noted as a single coding unit. Each pedagogical reasoning episode elicited the teachers' reasoning through discussion (Bannister, 2015), and these analysis units enabled the reduction of all collegial discussions into teacher talk episodes regarding students. To better identify these episodes, all utterances from four meetings of two teacher groups (i.e., sessions B and D) were transcribed, which enabled the easy identification of the episodes and student labels (student/s, some, mine, them, children, and us). These labels were then used to identify episodes in the video materials for the collegial discussion in seven other groups.

The abductive process was employed to develop the analytical framework (Bryman, 2016). A total of 109 episodes of meaningful utterances regarding the students' mathematical capabilities were identified. Specifically, teachers discussed their students in a positive/neutral manner in 22 episodes, and teachers characterized student capabilities by focusing on students' difficulties in mathematics learning in 87 episodes.

Table 2. Illustration of one episode of pedagogical reasoning using the coding scheme.

Episode	Type of difficulty	Attribution of blame or causality	Attribution of support	Comments
M: But I think a lot of these problem-solving tasks … I feel like it's too difficult. So, it's good for us to think maybe, but it's not … Not for my students, it's too difficult. C: Yes, it is too difficult. N: Is it too difficult? M: Yes, too difficult, yes. F: Yes. J: But I think they would fix it if they could talk, work together on it, think about it, and maybe, well, in some cases discuss their way through. It's so hard to tell everyone they would solve it themselves without working together.	Difficulties with problem-solving tasks	Students' cognitive abilities	Arrange students in groups to discuss and work together	This type of difficulty is specific (related to students' ability to work with problem-solving tasks). The type of support is relatively general (students should discuss, but they do not mention what or how such discussion should be).

With reference to Jackson et al. (2017), the focus of the subsequent analysis was the 87 episodes in which the teachers framed their students' difficulties in mathematics learning, which were also analyzed in line with the Swedish teaching practice. The teachers' views of students' difficulties in mathematics learning were associated with the instructional reforms (Anthony et al., 2018). Moreover, the study focused on the relationship between the teachers' views of students' capabilities and Swedish teaching practice to elucidate the effects of PD.

Table 2 presents a coding scheme that was inspired by diagnostic and prognostic framings (Anthony et al., 2018; Jackson et al., 2017) and was developed for transcript analysis. The teacher episodes focused on difficulties presented two main dimensions, namely, the type of difficulty and attribution of blame (diagnostic framing) and the type of support and attribution of support (prognostic framing).

In an early phase of categorization, the current study developed two main categories based on teachers' framing of students' difficulties in mathematics learning, namely, mathematics and discussion and reasoning. However, we found that "mathematics" is extremely broad and general. When classifying the findings within this category, we found other types of categories, such as problem solving, conceptual understanding, and procedural fluency. Together with discussion and reasoning, which we changed to reason and communication, we found that these categories relatively overlap with those of Kilpatrick et al. (2001) and their five strands of mathematical proficiency. The categories were then assigned to each pedagogical reasoning unit based on the teachers' framing of students' difficulties in mathematics learning. The majority of the categories were related to the five strands of mathematical proficiency (Kilpatrick et al., 2001), which inspired the Swedish mathematics curriculum (Skolverket, 2011), namely, (1) formulate and solve problems using mathematics and evaluate the selected strategies and methods, (2) use and analyze mathematical concepts and the connections between concepts, (3) select and use appropriate mathematical methods for calculations and for solving routine tasks, (4) conduct and follow mathematical reasoning, and (5) use mathematical forms of expression to discuss, argue, and report on issues, calculations, and conclusions.

From the recognition of these strands in the teachers' framing of students' difficulties in mathematics learning, the current study identified six categories, namely, students' problem-solving abilities (13 episodes); difficulties in reasoning and communicating using mathematics (21 episodes); difficulties with motivation, engagement, and courage (19 episodes); difficulties with conceptual understanding (17 episodes); difficulties with procedural fluency (9 episodes) and diverse abilities (4 episodes); and left-over categories labeled as other difficulties (4 episodes).

To enhance the reliability of this research, the first author identified 87 episodes. A random selection of 20 episodes, including those that the first author found difficult to categorize, was shared with the second author. These episodes were discussed and compared with respect to content, such as type of category, difficulty, and support. The agreed numbers for the type of category, difficulty, and support were 16, 19, and 17, respectively. After a thorough discussion, cases of disagreement were further discussed. In the next phase of the analysis, 2–4 episodes within each of the six categories were presented and discussed with a research team involving five other members familiar with the data set. Discrepancies in the coding were then discussed by the research team and were resolved until a consensus was reached. On the basis of these refinements of the coding scheme, the first author initiated the process of refining the analysis of the 87 episodes.

Prior to presenting the findings, we point out several limitations of the method. We examined the collegial discussions of teachers to understand their characterization of students' difficulties in mathematics learning. First, data were videotaped from two meetings of each group, which provides a snapshot of teachers' views of students' mathematical capabilities during the PD program Boost for Mathematics. For this reason, we are unable to give grounds whether teachers changed their characterization of the students' difficulties in mathematics learning through the PD program. Second, we collected statements from groups of teachers. Elliott et al. (2009) argued that when teachers meet for collegial discussions, one social norm is that the participants tend to listen respectfully to one another and exchange ideas. As such, less questioning among teachers occurs during collegial discussions. Thus, whether the group represents the view of individual teachers remains unclear. The third limitation is derived from the coding process. The first author coded all narratives in the data set, whereas the second author coded a subset of the total data set used to establish reliability. The final analysis was conducted by the first author. Thus, the proportion of the common coding of the data set could be higher or could involve several coders from the research team.

Results

This section provides an elaborate discussion of how the nine groups of teachers characterized students' difficulties in mathematics learning during the PD program for mathematics education. Table 3 presents the overview of the key findings for the diagnostic and prognostic teacher framings of students' difficulties in mathematics learning. The cognitive abilities of the students were the dominant diagnostic framing, except for the category *motivation, engagement, and encouragement*. In this category, the teachers frame the difficulty as related to their instruction. Prognostic framing, which highlights the types of solution, is dominant among teachers organizing and engaging

Table 3. An overview of the diagnostic and prognostic types of framing.

	Problem solving	Reason and communicate	Motivation, engagement, and encouragement	Conceptual understanding	Procedural fluency	Diverse abilities
Diagnostic framing	Students' cognitive abilities	Students' cognitive abilities or lack of experience	Type of instruction	Students' cognitive abilities	Students' cognitive abilities	Students' cognitive abilities
Prognostic framing	Students working in pairs or working more specifically with problem-solving tasks	Teachers engage students to discuss in groups or with the entire class	Variation in teaching	Teachers lowering the cognitive demand	None	Teachers categorize students into homogeneous groups

students to discuss in pairs or groups. Two categories differ, namely, *procedural fluency*, in which a lack of suggested solution exists, and *motivation, engagement, and encouragement*, where the suggested solution is a variation in teaching.

Framing students with problem-solving difficulties

The teacher groups were relatively concerned with problem solving and the challenges faced by the students, which were discussed in 13 episodes. Two strands of difficulty were expressed, namely, students were too solution focused (which resulted in poor reasoning and reluctance to solve problems using different methods) and students with difficulties formulating a similar task in relation to problem solving. The following excerpt is an example of a discussion on the first strand by teachers from Group 1, which discussed an instructional task:

> Plan a lesson where students will formulate their own problems. You should encourage your students to formulate their own problems based on one of the ways described in the text "formulating problems." Both ways, "formulate a similar problem" and "formulate a problem," must be represented in the teacher group.

1 Mia	It was really difficult. But it is also difficult when you lack experience. And then they got
2	the next question.
3 Helen	Haha. It was not clear for us.
4 Mia	You got all confused, and then it is difficult to imagine the students answers and how I
5	should proceed. You have to be flexible. Then they got the next question how many coins
6	and what could the value of the coins be. So, we reminded them again, remember now, it could
7	be one-krone coins, five-krone coins, and ten-krone coins. But then they fixated on the coins
8	being of only one denomination, and some drew only one-krone coins, some drew only
9	ten-krone coins. So I ended up putting the problem on the board and asked them to imagine
10	if they had a five-krone coin there and a one-krone coin there, but they could not follow this
11	argument.

The objective of the assignment was for students to suggest several solutions to an open problem regarding coins in a tin. In the lesson to which the teacher referred, the students needed to first guess how many coins were in the tin. However, the students found difficulty in solving the problem using coins with different values, which the teachers suggested was due to their lack of experience with coins and the value of the coins (1). The next task was to suggest the number of coins and their value (5–6). Moreover, the teachers perceived this task as extremely difficult for students. The explanation for this difficulty (type of problem) was that the students were fixated (7), that is, they only focused on the same type of coin. Maria, therefore, opted to discuss various solutions on the board (type of solution) but considered this review difficult for the students (8–11).

As also noted within this category and in many of the cases across categories, the teachers did not provide a thorough explanation of the type of support. Moreover, less argument occurred as the teachers tended to agree regarding the type of support, as demonstrated by the following case from Group 3:

1 Clara	But eh, how should we do, how could we help them in a proper way if you
2	do say that they should dare to think freely?
3 Brenda	But I feel that I need to put them together so they can create together.
4 Rita	Ah, I do also believe that this is not a task to work on individually.
5 Brenda	So they are put along the track with someone else and in a way get into it.
6 Clara	Maybe not in that large a group but instead perhaps two or …
7 Brenda	Two because ah
8 Kim	As if a couple
9 Rita	Two, maybe three

This group discussed how to support students with difficulties in formulating their problems during problem-solving tasks (1–2). They suggested that the students should work in groups (3) and argued that the students would have a better opportunity by working in groups (3) instead of working individually (4). As illustrated in this episode and common to nearly all episodes, the teachers did not discuss how to intervene when students interact. For instance, the teachers did not discuss how the students could draw connections from their collective invented strategies and representations when addressing a problem. Moreover, the teachers failed to design, initiate, and follow up learning opportunities based on their mathematical misconceptions of the students.

Within this category, the teacher's dominant diagnostic and prognostic framings were unproductive. Moreover, teachers' diagnostic framings of problem solving were mainly related to the cognitive abilities of students, which are indicated by their view that problem solving was extremely difficult for the students. The arguments are that the students are very solution oriented, lacked the ability to work with problem solving, and solve problems using multiple strategies. The suggested types of support illustrated in these episodes seemingly indicate that the teachers believed that the best method for addressing students' difficulties in mathematics learning was (a) to demonstrate similar problem-solving tasks, which is a more traditional teacher practice, or (b) to enable students to take responsibility for their learning and to work with their peers. In teachers' discussions, the role of the teacher also emerges. In an ongoing study on the role of the teacher reveals that teachers often take an invisible role. This means a teacher who avoids disturbing the students in their interactions about mathematics, only responding to students' questions.

Framing the reasoning and communication difficulties of students

Discussions regarding the reasoning and communication difficulties of the students were the most common (n = 21). These difficulties were explained by either a lack of experience or cognitive capability, which could be addressed by letting the students work in groups or develop their opportunities to talk regarding their mathematical thinking and explain their ideas together with the teacher, as exemplified in the following narrative from Group 1.

1 Hannah	They think it is too difficult, to explain their reasoning because they are so
2	concerned about the right answer. Because, when I read the problem to them
3	they raise their hands immediately and say they know the answer. Yes, but I have
4	said many times that I do not want the answer, I want to know how you think and
5	you should work together and cooperate. Many students have not an idea of what to do.
6 Helen	They always ask the teacher what to do.
7 Mia	I understand exactly what you mean. It is hard for the students to reason and communicate.
8 Hannah	Absolutely.
9 Mia	What could we do? How should we help them? In a proper way, so they are able to reason.
10 Helen	I think we need to put them in groups, to reason together.
11 Hannah	Yes, I think that too.
12 Mia	They will monitor someone else and be able to get into the reasoning.

This excerpt typical illustrates the results within this category. The teachers are concerned regarding establishing a classroom community where students can reflect and share ideas. Moreover, the teachers found difficulty in initiating a classroom practice that entails students to share their reasoning. They blame the lack of experience (1–2) and cognitive abilities (3–4) of the students (i.e., "the students do not have an idea of what to do"). Nevertheless, when these teachers are concerned regarding the lack of experience of students, they do not highlight the teacher or learning environment as a decisive factor. For instance, they do not discuss whether they should establish norms through which students can communicate and share ideas. The suggested type of support is to let students work together and cooperate (10). Thus, an underlying idea that students will gain more opportunities to discuss when they cooperate with peers exists. In terms of the type of support for mathematical challenges, teachers are relatively lacking in the manner in which they illustrated such types of support. Furthermore, analysis revealed episodes in which the teachers' descriptions of instructional actions to take were focused exclusively on reducing the rigor of reasoning required for students with difficulties. The reason is that teachers were frustrated by the lack of discussion and cooperation among student groups, which they believed was due to the ages of the students.

Within this category, the teachers expressed an unproductive diagnostic framing of students' difficulties in reasoning and communicating. In other words, they blame the students' lack of ability and experience and do not emphasize instructional opportunities, such as classroom norms, as the attribution of the blame. By contrast, the prognostic framing of the teachers was productive, whereas the type of solutions was to engage students in discussing in groups to encourage all students to participate in mathematical activities.

Framing the difficulties of students with motivation, engagement, and encouragement

A total of 19 episodes centered on the difficulties of students with motivation, engagement, or encouragement. This focus is in contrast with those of other episodes, which were more related to instruction and less to the students, that is, these difficulties were clearly viewed as the responsibility of the teachers. The most common support identified was task variation, as exemplified in the following excerpt from Group 6.

1 Clare	I was thinking about what you said Jimmie that the students were not engaged and it
2	may not be so easy to find tasks to create and I believe that you need time to create
3	tasks to encourage students to have the will to engage. I believe that too. Because
4	I was thinking of the box of matches, they were very engaged.
5	You have to work to find tasks that generate positive thinking and a will to work
6	further.
7 Sofia	And when you do that, you often need to do it together with someone else. To discuss
8	with someone else, what should we do to get into this.

Clare refers to Jimmie, who said that his students' lack of engagement in mathematics (1) was difficult for him. Clare then suggested finding tasks that increase student engagement (2–3) and referred to an experience with a digital program (4), that is, matchbox equations, which is presented in the guide for Boost for Mathematics. Therefore, the attribution of blame was placed on the lesson content and tasks given to students. The type of support suggested by Clare (5–6) was to find tasks that motivated students to work harder. Sofia, the coach, agreed but added that teachers needed to cooperate to find and develop such tasks (7–8).

There was a common perception in nearly all episodes, wherein the students' lack of motivation and engagement concerned the whole class, as illustrated in line 3, when they referred to the *students*. Therefore, the teachers did not relate this problem to the individual abilities or age of the students. They described possible support by finding suitable tasks to overcome the lack of motivation and engagement of the students, as illustrated in lines 2–3, which frequently involved a variation in teaching or providing students with open tasks with multiple answers. A type of support described was giving students "good" tasks. However, they did not elaborate on which tasks are good. As mentioned in Section 4.1, the teachers' suggestions for support were relatively general and with a few specific suggestions regarding the types of activities or tasks that could be used to overcome student engagement problem.

In contrast to other categories, the teachers' dominant diagnostic and prognostic framings were productive in terms of the difficulties of students with motivation, engagement, and encouragement. The type of problem was related to instructional opportunities as a variation between teaching and suitable tasks. The type of support proposed is engaging and motivating students through tasks that present the possibility of involving all students in solving challenging problem-solving tasks.

Framing the difficulties of students with conceptual understanding

The teachers mainly portrayed the difficulties of students in terms of conceptual understanding by emphasizing that students did not understand or they had difficulties with mathematical words and concepts. Diagnostic framing was aligned to the student's lack of ability to grasp concepts. In these episodes, prognostic framings were frequently associated with the actions of the teachers, as exemplified in the excerpt from Group 5. The teachers discussed the module "language in mathematics" and requirement for preparing a lesson, in which students could work with translations of different parts of mathematical language using concept maps. In the excerpt below, Sandra reflected on her lesson.

Sandra	I made one (concept maps) partly with algorithms and partly with time, and they had to sort it out. Eh, I discovered that this was difficult. It was several words I thought they knew which they had great difficulties to explain and they did not know. So I ended up with funneling, a lot of funneling (laughter). So it ended up that we did it together instead, and I feel that I need to work a lot with this, but they have difficulties; these students and words are difficult.
Alice	Exciting.
Sandra	Yes, it is difficult.

The student groups were required to organize different concepts related to time and arithmetic operations (1). Sandra found that this task was very difficult for her students (2), which was related to explaining the concepts (3) and a lack of understanding of these concepts (3, 6, and 8). Initially, Sandra assumed that her students were familiar with those concepts. Sandra described support in two steps. First, she guided the students as they worked in groups (4) but ended up working on the problem together with the students in a whole class setting (5).

Within this category, the teacher's dominant diagnostic and prognostic framings were unproductive for students with difficulty in conceptual understanding, which were attributed to their lack of cognitive abilities. The type of support was to reduce the rigor of the learning goals for students with difficulties. Such support required the use of funneling patterns to explain the connection between concepts.

Framing the difficulties of students with procedural fluency

In terms of conceptual understanding, the teachers recognized the difficulties of students with procedural fluency, which they suggested are related to the inherent traits of students, that is, they felt that their students lacked the required capabilities. However, this category differed from other categories in terms of prognostic framing. When the teachers portrayed difficulties in students' procedural fluency, nearly all episodes lacked a description of the type of support. Instead, they focused on other features, as exemplified in the following narrative from Group 7.

Tracy	I asked them to add all the fractions to see how many whole numbers we end up? Some managed to add two halves but had problems with the quarters and five twenties. It was quite funny.
Gina	That was good.
Tracy	Yes, but actually one person was able to calculate, she understood. The others, not that much. No, it was a little too difficult.
Clara	But was everyone active?
Tracy	Everyone was active.
Clara	Do you know what made them active?
Tracy	Yes, I think I was able to engage these students who normally get involved in such tasks, as they also could draw if they wanted.

The teacher was referring to a lesson in which the students worked in groups to represent fractions using different notations. Each group worked on a specific fraction to which they were assigned. One of the objectives of the lesson was to understand that fractions of the same size could be written in different notations. Therefore, the groups worked with fractions, such as $\frac{1}{2}$, $\frac{2}{4}$ and $\frac{5}{10}$. In the next phase of the activity, they arranged the fractions in a sequence to decide which fractions had the same size. The teacher then asked the students to add the fractions (1–2). The teacher considered this task very difficult for the students as only one student understood how to add fractions with different denominators (5–6). Thus, Clara opted to focus on something distinctive by asking if the students were active (7), which tended to emphasize the students' activeness instead of their ability to add fractions (7–11).

Framing the difficulties of students with diverse abilities

The discussion on students with diverse abilities was frequently conducted in connection with the social processes in the classroom and focused on all students being involved in group discussions during group work and especially on challenges faced by students with low levels of abilities when working with other students within a group, as exemplified in the following excerpt from Group 8:

1 Karen	Just now when we discussed about how to include all, it is a bit exciting
2	to see what is happening if we put them together in different ways or randomly. I
3	find this exciting. You do know your student groups so you have to think what is
4	suitable for you.
5 Yvonne	But if you have a low achiever who does not talk that much and are put together
6	with a high-achiever then they will not say anything.
7 Anna	I do not think so either.
8 Marie	No
9 Yvonne	I do believe that they will not say anything.
10 Helen	No.
11 Yvonne	They will not say anything.
12 Karen	This is something that comes with age. This is why it is important that you know
13	your groups and what is working.

The teachers discussed different methods for grouping students. One suggestion was to randomly group the students, as suggested by the coach (2–3). Yvonne contradicted this suggestion by saying that low-ability students would not say anything in a discussion with high-ability students (5–6 and 11). All other teachers agreed (7, 8, and 10). The coach, Karen, replied that the other teachers are free to decide on how to group their students (12–13).

The framing of the teachers regarding the type of problem is unproductive, where student difficulties are attributed to inherent traits. Several students are better than others in terms of mathematical abilities and in participating in group discussions. The teachers seemingly desire that all students should participate in rigorous mathematical group discussions, which is a productive prognostic framing, but lack specific actions on how to achieve this goal.

Summary of findings

During the discussions, diagnostic framing was mostly related to inherent traits or lack of ability, which was especially prominent when the teachers discussed difficulties in conceptual understanding and procedural fluency. However, the teacher framings for difficulties in motivation, engagement, and encouragement were mainly related to instruction. In other words, the teachers took a diagnostic productive view.

The type of support or solutions to these difficulties was generally formulated. The teachers offered three main solutions, namely, grouping students to encourage them to take responsibility and overcome their difficulties, engaging the entire class with the mathematical idea, and lowering the cognitive demand of the activity. The teachers were frequently concerned with the abilities of the students to discuss and solve problems in groups and stressed that students could learn from one another. This solution is seemingly standard to the teachers. As such, few specific practices were suggested to overcome the struggles of students with learning difficulties. Moreover, teachers considered taking the mathematical solutions of the students as a starting point when describing their prognostic support.

Discussion and conclusion

This study contributes to research on teachers' collegial discussions in PD programs on the difficulties of students related to mathematics learning (Anthony et al., 2018; Horn, 2007; Jackson et al., 2017; Russo et al., 2020). Analysis focused on the framing of teachers regarding the abovementioned difficulties in terms of source (diagnostic) and support (prognostic). This study adds to the field by highlighting the teachers' views of students' difficulties in mathematics learning in collegial discussions, especially by categorizing difficulties related to mathematical proficiency (Kilpatrick et al., 2001). Below we outline and discuss the findings and contributions of the study urging the reader to interpret them in the light of the relatively large number of teachers participating in the study but also in relation to the potential limitations of the study elaborated on in the section of methodology.

The findings illustrate that teachers hold different productive and unproductive views of such difficulties compared with mathematical proficiency. Another contribution concerns the value of collegial discussions for eliciting the framing of teachers. The study investigated the framing of students' difficulties in mathematics learning during conversations with colleagues. Although framing was not a part of the content in the PD program, the group of teachers framed the students' difficulty in mathematics learning to a certain extent. Therefore, this study concludes that teachers are concerned regarding the difficulties of students in mathematics learning when discussing mathematics teaching and learning.

The three key findings of the study are related to the productive and unproductive framings of students' difficulties in mathematics learning, teachers' concerns regarding their students' discussion, and reasoning, which were observed in approximately 25% of all episodes. The teachers' framing of their students' difficulties in mathematics learning as being common to all students contradicts the finding of Gamoran Sherin and Van Es (2009), where teachers referred to students as being *low* or *high* when explaining student performance.

Bannister (2015) and Jackson et al. (2017) used the productive and unproductive diagnostic and prognostic framing concepts to explain how teachers signal encouragement or discouragement when taking action for students with difficulties. Bannister (2015) found that articulating the unproductive type of the explanations of students' difficulties in mathematics learning is concurrent with articulating the unproductive type of support. Moreover, productive explanations tend to correspond to articulating productive support. Although a strong relationship may exist between the diagnostic and prognostic framings of teachers, these relationships were not entirely straightforward, as demonstrated by Jackson et al. (2017) and the current study. However, the results of the study are in contrast with those of Bannister (2015), where teachers tended to describe unproductive diagnostic framing, such as "it was too difficult for the students," in connection with a productive prognostic framing, such as working with open-ended tasks and facilitating students in discussing the problem in groups.

Analysis revealed that the teachers were mainly concerned regarding six difficulties, namely, problem solving, reasoning and communicating regarding mathematics, motivation and engagement, conceptual understanding, procedural fluency, and diverse abilities. Nearly 25% of the episodes were focused on difficulties associated with reasoning and communicating regarding mathematics, which may be due to the fact that the Swedish curriculum emphasizes mathematical reasoning and argument as a part of the need to foster democratic citizenship and to develop democratic mindsets and skills (Forsberg et al., 2016). The teachers suggested that the best ways to overcome these difficulties working through those problems with other students are to provide support through class discussions, provide students with cognitively demanding tasks, or lower the cognitive demand by offering simple tasks.

Analysis further revealed that the teachers appeared to view their students as those that share equal opportunities to engage in mathematics discourse. These results were in contrast to the view that teachers typically view cognitively demanding instruction modes as inappropriate for low-achieving students (Horn, 2007; Jackson et al., 2017). The teachers discussed the difficulties of students as a whole instead of as individual problems or problems related to differences between high- and low-performing students. When they discussed students with low-ability, the solution to help these students to overcome difficulty in group participation was to group students with low-ability together. However, this concern was mainly related to the need of students to participate in discussions instead of a focus on mathematical content. A possible reason is that the teachers mainly viewed engagement and motivation as important for encouraging students to continue working instead of referring to task-based approaches to overcome difficulties in mathematics.

Designers of PD programs for mathematics education could benefit from the findings of the present study. Although the sample teachers tended to rely on productive prognostic framing to support students with difficulties, they frequently described support in a general manner. For example, they suggested that the students work with peers but did not elaborate on how to provide students with the knowledge to learn to engage in such forms of discourse. Furthermore, they suggested that students work on cognitively demanding tasks, such as problem-solving or open-ended tasks. However, they did not specify the tasks that could help students overcome such difficulties. At times, the teachers used unproductive diagnostic framing by blaming student difficulties on inherent traits. When teachers first engaged in professional discussions regarding the students, they focused on what students cannot do (Gresalfi & Cobb, 2011; Kazemi & Franke, 2004). However, professional learning should focus on raising the expectations of teachers regarding their students and on how teachers can enable the students to meet such expectations.

This study examined the teachers' framing regarding students with difficulties. However, further research should focus on how collegial discussions can develop more productive means of explaining student difficulties. This necessary shift in teachers' views can be guided through key aspects related to teaching and learning mathematics, such as building on students thinking or using cognitively demanding tasks to overcome specific difficulties in mathematics. The current study found that teachers frequently use productive prognostic framing in a general manner. Therefore, further studies should focus on the functions of prognostic framing based on research instead of classroom-based experience. The current study is a snapshot of teachers' views of students' mathematical capabilities during the PD program entitled Boost for Mathematics. Thus, further research could address changes in the teachers' framing of the difficulties experienced by students in mathematics learning during such a PD program.

Regarding the role of collegial discussion and teacher's pedagogical development, the implication for future research and practice would be the focus on teachers' productive prognostic framings. In line with previous findings (Bannister, 2015; Jackson et al., 2017), teachers need assistance in terms of supporting students faced with difficulties in participating in rigorous mathematical activities. The current study demonstrated that the framing of teachers regarding support for students with difficulties is relatively general.

Note

1. For more information of the research project, see https://www.mdh.se/en/malardalen-university/research/research-projects/theorizing-teacher-use-of-curriculum-material-within-mathematics-classroom-practice-

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References

Anthony, G., Hunter, R., & Hunter, J. (2018). Challenging teachers' perceptions of student capability through professional development: A telling case. *Professional Development in Education*, *44*(5), 650–662. https://doi.org/10.1080/19415257.2017.1387868

Bailey, D. H., Watts, T. W., Littlefield, A. K., & Geary, D. C. (2014). State and trait effects on individual differences in children's mathematical development. *Psychological science*, *25*(11), 2017–2026. https://doi.org/10.1177/0956797614547539

Bannister, N. A. (2015). Reframing practice: Teacher learning through interactions in a collaborative group. *Journal of the Learning Sciences*, *24*(3), 347–372. https://doi.org/10.1080/10508406.2014.999196

Boaler, J. (2015). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages and innovative teaching*. John Wiley & Sons.

Boesen, J., Helenius, O., Bergqvist, E., Bergqvist, T., Lithner, J., Palm, T., & Palmberg, B. (2014). Developing mathematical competence: From the intended to the enacted curriculum. *The Journal of Mathematical Behavior*, *33*, 72–87. https://doi.org/10.1016/j.jmathb.2013.10.001

Boesen, J., Helenius, O., & Johansson, B. (2015). National-scale professional development in Sweden: theory, policy, practice. *ZDM*, *47*(1), 129–141. https://doi.org/10.1007/s11858-014-0653-4

Bryman, A. (2016). *Social research methods*. Oxford university press.

Cobb, P., & Jackson, K. (2011). Towards an empirically grounded theory of action for improving the quality of mathematics teaching at scale. *Mathematics Teacher Education and Development*, *13*(1), 6–33.

Cobb, P., Jackson, K., Henrick, E., & Smith, T. M. (2018). *Systems for instructional improvement: Creating coherence from the classroom to the district office*. Harvard Education Press.

Desimone, L. M. (2009). Improving impact studies of teachers' professional development: Toward better conceptualizations and measures. *Educational Researcher*, *38*(3), 181–199. https://doi.org/10.3102/0013189X08331140

Desimone, L. M., & Garet, M. S. (2015). Best practices in teachers' professional development in the United States. *Psychology, Society, & Education*, *7*(3), 252–263. https://doi.org/10.25115/psye.v7i3.515

Dweck, C. S. (2000). *Self-theories: Their role in motivation, personality, and development*. Psychology press.

Elliott, R., Kazemi, E., Lesseig, K., Mumme, J., Carroll, C., & Kelley-Petersen, M. (2009). Conceptualizing the Work of Leading Mathematical Tasks in Professional Development. *Journal of Teacher Education*, *60*(4), 364–379. https://doi.org/10.1177/0022487109341150

Forsberg, E., Hortlund, T., & Malmberg, K. (2016). The assessment culture of school leadership. *Nordic Studies in Education*, *36*(2), 141–158. https://doi:10.18261/issn.1891-5949-2016-02-05

Franke, M. L., Kazemi, E., & Battey, D. (2007). Mathematics teaching and classroom practice. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (2nd ed., pp. 225–256). Information Age Publishing.

Gamoran Sherin, M., & Van Es, E. A. (2009). Effects of video club participation on teachers' professional vision. *Journal of teacher education*, *60*(1), 20–37. https://doi.org/10.1177/0022487108328155

Garet, M. S., Wayne, A. J., Stancavage, F., Taylor, J., Eaton, M., Walters, K., Song, M., Brown, S., Hurlburt, S., Zhu, P., Sepanik, S., & Doolittle, F. (2011). *Middle school mathematics professional development impact study: Findings after the second year of implementation*. Washington, DC: U.S. Department of Education. Retrieved from http://ies.ed.gov/pubsearch/pubsinfo.asp?pubid=NCEE20114024.

Goffman, E. (1974). *Frame analysis: An essay on the organization of experience*. Harvard University Press.

Goodwin, C. (1994). Professional vision. *American Anthropologist*, *96*(3), 606–633. doi:10.1525/aa.1994.96.3.02a00100

Gresalfi, M. S., & Cobb, P. (2011). Negotiating identities for mathematics teaching in the context of professional development. *Journal for Research in Mathematics Education*, *42*(3), 270–304. https://doi.org/10.5951/jresematheduc.42.3.0270

Horn, I. S. (2007). Fast kids, slow kids, lazy kids: Framing the mismatch problem in mathematics teachers' conversations. *The Journal of the Learning Sciences*, *16*(1), 37–79. https://doi.org/10.1080/10508400709336942

Jackson, K. (2011). Approaching participation in school-based mathematics as a cross-setting phenomenon. *Journal of the learning sciences*, *20*(1), 111–150. https://doi.org/10.1080/10508406.2011.528319

Jackson, K., Gibbons, L., & Sharpe, C. J. (2017). Teachers' Views of Students' Mathematical Capabilities: Challenges and Possibilities for Ambitious Reform. *Teachers college record*, *119*(7), 1–43. doi:10.1177/016146811711900708

Jackson, K., Wilhelm, A. G., & Munter, C. (2018). Specifying goals for students' mathematics learning and the development of teachers knowledge, perspectives, and practice. In P. Cobb, K. Jackson, E. Henrick, & T. M. Smith (Eds.), *Systems for instructional improvement: Creating coherence from the classroom to the district office* (pp. 43–64). Harvard Education Press.

Kazemi, E., & Franke, M. L. (2004). Teacher learning in mathematics: Using student work to promote collective inquiry. *Journal of mathematics teacher education*, *7*(3), 203–235. https://doi.org/10.1023/B:JMTE.0000033084.26326.19

Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping children learn mathematics*. National Academies Press.
Lahann, P., & Lambdin, D. V. (2020). Collaborative Learning in Mathematics Education. In S. Lerman (Ed.), *Encyclopedia of Mathematics Education* (pp. 75–76). Springer.
Lindvall, J., Helenius, O., & Wiberg, M. (2018). Critical features of professional development programs: Comparing content focus and impact of two large-scale programs. *Teaching and Teacher Education*, *70*, 121–131. https://doi.org/10.1016/j.tate.2017.11.013
Munter, C. (2014). Developing visions of high-quality mathematics instruction. *Journal for Research in Mathematics Education*, *45*(5), 584–635. https://doi.org/10.5951/jresematheduc.45.5.0584
Munter, C., & Wilhelm, A. G. (2021). Mathematics teachers' knowledge, networks, practice, and change in instructional visions. *Journal of Teacher Education*, *72*(3), 342–354. https://doi.org/10.1177/0022487120949836
National Council of Teachers of Mathematics (NCTM). (2000). *Principles and standards for school mathematics*. National Council of Teachers of Mathematics.
Rittle-Johnson, B., Siegler, R. S., & Alibali, M. W. (2001). Developing conceptual understanding and procedural skill in mathematics: An iterative process. *Journal of Educational Psychology*, *93*(2), 346–362. https://doi.org/10.1037/0022-0663.93.2.346
Russo, J., Bobis, J., Downton, A., Hughes, S., Livy, S., McCormick, M., & Sullivan, P. (2020). Elementary teachers' beliefs on the role of struggle in the mathematics classroom. *The Journal of Mathematical Behavior*, *58*, 100774. https://doi.org/10.1016/j.jmathb.2020.100774
Ryve, A., & Hemmi, K. (2019). Educational policy to improve mathematics instruction at scale: conceptualizing contextual factors. *Educational Studies in Mathematics*, *102*(3), 379–394. https://doi.org/10.1007/s10649-019-09887-6
Ryve, A., Hemmi, K., & Kornhall, P. (2016). *Skola på vetenskaplig grund [School on a scientific basis]*. Natur & Kultur.
Skolverket. (2011). *Läroplan för grundskolan, förskoleklassen och fritidshemmet*. Skolverket.
Skolverket. (2018). Matematiklyftet [Boost for Mathematics]. Retrieved from https://larportalen.skolverket.se/#/moduler/1-matematik/alla/alla.
Sun, K. L. (2019). The mindset disconnect in mathematics teaching: A qualitative analysis of classroom instruction. *The Journal of Mathematical Behavior*, *56*, 100706. https://doi.org/10.1016/j.jmathb.2019.04.005
Wilhelm, A. G., Munter, C., & Jackson, K. (2017). Examining relations between teachers' explanations of sources of students' difficulty in mathematics and students' opportunities to learn. *The Elementary School Journal*, *117*(3), 345–370. https://doi.org/10.1086/690113
Wilson, P. H., Sztajn, P., Edgington, C., Webb, J., & Myers, M. (2017). Changes in teachers' discourse about students in a professional development on learning trajectories. *American Educational Research Journal*, *54*(3), 568–604. https://doi.org/10.3102/0002831217693801