International Journal of Science and Mathematics Education
https://doi.org/10.1007/s10763-022-10294-2

Teachers' Error-handling Practices Within and Across Lesson Phases in the Mathematics Classroom

Odd Tore Kaufmann[1] · **Maria Larsson**[2] · **Andreas Ryve**[1,2]

Received: 27 August 2021 / Accepted: 18 May 2022
© The Author(s) 2022

Abstract
Multiple studies have been conducted regarding teachers' error-handling practices, and how errors can be treated as opportunities for learning, albeit in the context of whole-class discussions. The aim of the present research is to continue to investigate teachers' error-handling practices as they occur in different phases of maths lessons: introduction of the task, when students are working alone, and when students are working in pairs and finally, as part of the whole-class discussion. The study included 51 lessons from twelve teachers. A cross-case analysis was made across the individual teacher cases to look for similarities and differences between different teachers' error-handling practices across the lesson phases in order to create teaching profiles with similar handling of student errors across the lesson phases. Five error-handling teaching profiles were identified; correcting errors throughout all phases, correcting errors during students' work while few errors are brought up in whole class, correcting errors during students' work while using a variety of practices in whole class, ignoring errors while using some of them in whole class, and discussing and explaining errors.

Keywords Conceptual errors · Error-handling profiles · Lesson phases · Teachers' error-handling practices

✉ Odd Tore Kaufmann
odd.t.kaufmann@hiof.no

Maria Larsson
maria.larsson@mdu.se

Andreas Ryve
andreas.ryve@mdu.se

1 Faculty of Teacher Education and Languages, Østfold University College, Halden, Norway

2 Mälardalen University, Västerås, Sweden

Published online: 20 June 2022

Introduction

About two decades of research has been dedicated to exploring how teachers handle student errors as part of their classroom interactions. Errors can provide opportunities for learning if the reasoning and underlying errors can be identified and explored as part of learning activities (Brodie, 2014). Alternatively, certain error-handling practices could also influence students to develop unproductive mathematical identities (Gardee & Brodie, 2021). Due to several dimensions of the role of errors for students' learning, research has examined different strands of teachers' error-handling practices in mathematical teaching, such as how student errors can be treated as an opportunity for learning in the classroom (Ingram et al., 2015), how teachers from different cultures handle errors differently (Santagata, 2005; Schleppenbach et al., 2007), and how teachers' beliefs and knowledge influence their error-handling practices during whole-class discussions (Bray, 2011). In this article, we examine teachers' error-handling practices in different phases of mathematics lessons. That is, although research has investigated how teachers handle errors in whole-class situations (Bray, 2011; Ingram et al., 2015; Schleppenbach et al., 2007), limited attention has been paid to ascertain patterns of error-handling practices across different lesson phases. Moreover, there is still limited research on how teachers actually deal with learner errors in mathematics classrooms (Gardee & Brodie, 2015). The aim of the present paper is to add to this literature by answering the following research question: What characterizes teachers' error-handling practices of students' conceptual errors within and across different lesson phases? Based on Schleppenbach et al.'s (2007) and Santagata's (2005) frameworks, an abductive process was used to develop an analytical framework for teachers' error-handling practices. A cross-case analysis was conducted on teachers to look for similarities and differences between different teachers' error-handling practices across the lesson phases. The analysis of twelve teachers in 51 lessons enable us to characterize patterns of error-handling practices in different lesson phases as well as to identify teaching profiles of how teachers handle errors across different lesson phases.

Research background

The research background is presented in three parts. The first part describes what is considered as an error in the mathematics classroom. The second part presents the role errors can take in the classroom, and how errors can be used as an opportunity for learning. The third part describes different ways teachers' responses to errors have been categorized in the existing research.

What is considered an error in the mathematics classroom?

The concept of an error is used in research with slightly different meanings and used interchangeably at times with related concepts such as misunderstanding, slip, and mistake. Gardee and Brodie (2015, 2021) distinguish between errors and slips. Errors occur at a deeper conceptual level than slips (which are caused by carelessness). Errors are systematic and caused by misconceptions, often due to overgeneralizations of correct prior knowledge (Gardee & Brodie, 2015). In some studies, a distinction between conceptual errors and slips is not made (e.g., Bray, 2011; Ingram et al., 2015; Schleppenbach et al., 2007). We, however, expect that teachers would handle errors and slips differently, i.e., it would not make sense to have a long discussion or offer an explanation in relation to a slip since slips are easily corrected when pointed out. We, therefore, distinguish between slips and conceptual errors. As mentioned in the introduction and then further clarified in the methodology, this study focused on conceptual errors, which might be considered either misconceptions, or yet incoherent knowledge pieces that are productive in some situations but not in others (Gardee & Brodie, 2021; Scheiner, 2020). According to a knowledge-in-structures' perspective, students' misconceptions can be characterized as systematic patterns of errors (Scheiner, 2020). Alternatively, students' conceptual errors could be characterized as alternative conceptions that prove productive in some situations but not in others—a phenomenon that Scheiner (2020) refers to as a "knowledge-in-pieces perspective." These two perspectives give different but complementary accounts of conceptual errors (Scheiner, 2020). As Scheiner (2020) emphasizes, "the two perspectives give different accounts for knowledge development on different levels over different time spans" (p. 137). From our data, we could not decide whether a student error stems from a more stable misconception or from an alternative conception that is more contextual in nature. Yet, we believe that it is crucial to focus on the reasoning behind such conceptual errors in mathematics teaching, regardless of whether a student's conceptual error stems from a misconception, a not yet developed conception, or an alternative conception that is more contextual in nature (Brodie, 2014). Therefore, we analyze the teacher-student interaction in the episode that follows when a conceptual error occurs in the classroom (cf. Bray, 2011; Ingram et al., 2015; Schleppenbach et al., 2007).

The role of errors in teaching and learning mathematics

Student errors are a natural and inevitable part of all classroom interactions, and the mathematics classroom is no different. Although every maths teacher will experience his or her students making errors, recent research suggests that how teachers handle these errors in the classroom is essential for students' opportunity to learn (Brodie, 2014). The role of errors can be conceptualized in two distinct ways: as a problem to be avoided or as an opportunity for learning (Ingram et al., 2015; Swan, 2006). Influential steering documents, such as National Council of Teachers of Mathematics [NCTM] (2000), put forth the argument that teachers should develop their basis for teaching around a familiarity with the common difficulties and misconceptions encountered by their students.

From such a perspective, two major functions of errors in mathematics education are highlighted in the research literature: (1) the neurological point of view where, for example, Boaler (2015) argues that errors are particularly productive pathways for building new concepts and thus require students and teachers to develop dynamic mindsets and (2) errors as "springboards for inquiry" (Borasi, 1994; Bray, 2011) into mathematical concepts, which ensures that students are supported in mathematical investigations and have opportunities to explore, discuss, and justify their ideas. Discussing errors with the students will give teachers access to students' current thinking (Gardee & Brodie, 2015). Students might understand and learn better when errors are discussed in the classroom, rather than being corrected or avoided (Gardee & Brodie, 2021). If teachers avoid or ignore the error, the students will not have access to understand why their errors are incorrect. Further, when teachers correct the error, it might be the case that the teacher has identified and evaluated the error rather than interpreted the error from the learner's perspective (Gardee & Brodie, 2015).

Teachers' responses to errors

The existing research has categorized teachers' responses to errors (i.e., error-handling practices) in different ways: Santagata (2005), for example, compared teachers' error-handling practices in American and Italian classrooms and coded teachers' responses into ten categories; Schleppenbach et al. (2007) compared teachers' error-handling practices in American and Chinese classrooms and categorized these practices into two types of responses —teachers' statements and teachers' questions—which were subsequently coded into eleven categories. These categories from Santagata (2005) and Schleppenbach et al. (2007) will be further elaborated in the analytical framework section.

Bray (2011) analyzed teachers' error-handling practices in 16 math lessons, four per teacher, and categorized these practices using three dimensions: (1) intentionally focusing on erroneous solutions as part of whole-class discussions, (2) responding to students' errors in whole-class discussions in ways that promote conceptual understanding (e.g., teachers successfully interpret students' errors in the moment and formulate clear, conceptually based questions and explanations), and (3) mobilizing students as a community of learners when errors emerge in whole-class discussions (e.g., teachers engage students in addressing and discussing the errors). Handling of errors in ways that adhere to Bray's (2011) dimensions can be seen in contrast to teachers keeping a tight control of the discourse and limiting the extent to which students work together to judge and resolve mathematical errors.

In their examination of how one teacher in her Grade 9 classroom engaged with mathematical errors, Gardee and Brodie (2015) identified four kinds of different ways teachers interacted with learner errors: (1) ignoring, (2) correcting, (3) probing, and (4) embracing students' errors. Ignoring means that the teacher does not acknowledge or engage with learner errors. Correcting means that the teacher has identified and evaluated the errors rather than interpreted the errors from the learners' perspective. Probing means that the teacher attempts to understand how the errors make sense to learners, by asking follow-up questions. Embracing means that the teacher uses errors constructively to generate new knowledge for the learner who has made the error. Gardee and Brodie (2015) found that errors were ignored in about one third of the instances in which errors occurred.

The categorization of the errors from the research studies mentioned above could also be divided into private and public error-handling (Heinze, 2005). Bray (2011), Ingram et al. (2015), and Schleppenbach et al. (2007) focused on public error-handling practices as part of whole-class discussions. Santagata (2005) focused on the difference between private (i.e., only for the student who made the mistake) and public error-handling. She found that even if the nature of mistakes that occurred in Italian and US mathematics lesson were very similar, the two countries differed in the extent to which the activity of correcting mistakes in public versus private setting. In Italy, almost all mistakes (97%) were discussed publicly while in USA, about 61% of mistakes per lesson were discussed publicly. The focus was on the average percentages of mistakes per lesson discussed publicly versus privately, and Santagata did not specify the categorization of teachers' error-handling practices in public or private handling of the error.

In terms of what influences teachers' responses to errors in the classroom, the research literature has identified as contributing factors the classroom norms, the teachers' beliefs about mathematical learning and teaching and the teachers' mathematical knowledge (Bray, 2011; Ingram et al., 2015). Bray (2011) calls for further research to consider if teachers error-handling practices hold true across various settings. We elaborate on and contribute to what influences teachers' responses to errors in the classroom, to consider different phases of the lesson, and we build on Santagata's (2005) argument that teachers' error-handling can either be a public or a private activity.

Methodology

In order to capture teachers' responses to students' errors in the classroom, the goal of this study was to consider teacher-student communication when a student has said or written something that is not mathematically correct. The sections below describe the study context, data collection techniques, and analysis of how teachers respond to student errors.

Study context

To be able to explore how teachers handle conceptual errors in mathematics classroom, we collected data from the “Boost for Mathematics.” This program aims at supporting teachers in establishing ambitious teaching (Lampert et al., 2010; Skolverket, 2018). The key principles of ambitious teaching are, e.g., to focus on developing conceptual understanding of key mathematical ideas and to engage students in mathematical practices that include reasoning, problem-solving, and communicating mathematically. Teachers’ error-handling practices are not an explicit part of this program, but since “Boost for Mathematics” builds on ambitious teaching, student errors are an important role in mathematics lessons. Therefore, we find this program as an interesting case for teachers’ error-handling practices. Below, we describe more about the “Boost for Mathematics” program. In Sweden, the Swedish National Agency for Education launched the curriculum-based professional development program called “Boost for Mathematics” (Skolverket, 2018), which cost approximately 65 million euros. The aim of the program was to strengthen the quality of teaching and student performance and was carried out by almost 80% of Swedish mathematics teachers during the 2013/2014, 2014/2015, and 2015/2016 academic years.[1] The program was made up of 24 modules for teachers in compulsory school, eight modules per grade level (1–3, 4–6 and 7–9), which were developed to support teachers working in teams to plan, establish, and reflect on pedagogical practices in the math classroom. Each module focuses on certain mathematical content (i.e., number sense, algebra, geometry, and problem-solving), how students learn this content, and how teachers can support their learning. The focus is on the five mathematical competencies in the Swedish curriculum, which include problem-solving, reasoning, communication, conceptual understanding, and procedural fluency. The curriculum is distributed digitally via a web portal[2] and includes articles, instructions, and video films. Each module is designed to support groups of teachers within a given semester to engage in eight iterations containing individual preparations (session A), collegial discussions regarding the resources as well as planning lessons (session B), individual classroom teaching based on the planning (session C), and collective reflections on their classroom instruction (session D). Teachers are required to complete two modules, one module per semester, and the PD sessions are held at each school with the support of a trained coach who is also an experienced mathematics teacher. The goals for “Boost for Mathematics” is to increase students’ mathematical achievement by strengthening the instruction and supporting teachers’ implementation of the national mathematics curriculum (Skolverket, 2018).

[1] In this project, the authors constructed the data set for the 2015/2016 academic year.

[2] https://larportalen.skolverket.se/#/moduler/1-matematik/alla

Data collection

A large data set from the "Boost for Mathematics" program was constructed during the 2015/2016 academic year funded by the Swedish National Research Council. Mathematics teachers and coaches from several large Swedish municipalities teaching grades 1–9 were asked to participate in the data collection process if they were about to start the "Boost for Mathematics" program in the autumn of 2015. Fifty-one teachers in years 1–9 agreed to participate. The present paper is part of this larger research project. Only teachers ($N = 12$) from the fourth, fifth, and sixth grade were included in the present research; however, as they formed a large data set (51 lessons) making it feasible to analyze all the lessons within this grade span. The lessons studied form a part of the PD initiative and are focused on conceptual aspects of mathematics and problem-solving. As such, the data enables us to contribute to conceptualizations of teachers' error-handling practices in conceptually and problem-solving focused lessons, rather than practices in procedurally oriented mathematics lessons. The number of video-taped lessons for each teacher varied from two to six lessons spread over the academic year. Two video cameras and one sound recorder were used in the classroom to capture these interactions. One of the cameras was statically located at the back of the classroom, and the second camera was handled by a mobile research assistant following the teacher.

Analytical framework

An abductive process was used to develop the analytical framework for the present research (Bryman, 2016). Drawing upon Schleppenbach et al.'s (2007) and Santagata's (2005) frameworks, modifications were made to their categories to create eight categories in total (which will be discussed in more detail below). Developing the analytical framework has been a continuous process, moving back and forth between our data and the analytical framework, and gradually making distinctions between different categories clearer. We focus on whole episodes of teachers' error-handling practices. We started by categorizing teachers' error-handling practices with the type of discourse (non-interactional or interactional) and foci (non-conceptual or conceptual). Within these four combinations, four categories appeared (Table 1).

Table 1 Four of the categories of the analytical framework, with the aspects of interactivity, foci, and description

	Interactive/non-interactive	Foci
Direct correction	Non-interactive	Non-conceptual
Embedded correction	Interactive	Non-conceptual
Teacher explanation	Non-interactive	Conceptual investigation
Discussion	Interactive	Conceptual investigation

Direct correction and *teacher explanation* are constituted by a non-interactive type of discourse. *Direct correction* is non-conceptual as the teacher does not explain the underlying conceptual reasons for why it is not correct while *Teacher explanation* has a conceptual focus. These non-interactive practices stem from the previous literature. For example, *Teacher explanation* stems from Schleppenbach et al.'s (2007) category "providing explanation or direction." *Direct correction* stems from Brodie's (2013) category "correct errors," Santagata's (2005) category "Gives correction," and Schleppenbach et al.'s (2007) categories "Giving the correct answer" and "Telling the student the answer is wrong." Since the focus of this study is on conceptual versus non-conceptual handling of student errors, we chose not to divide direct correction in different subcategories, although some literature (e.g., Schleppenbach et al., 2007) make distinctions between correcting the error by giving the correct answer and telling the student that the answer is wrong so that the students themselves can correct it. Other scholars (e.g., Gardee & Brodie, 2015) do not make this distinction but frame it as the teacher correcting errors.

Embedded correction and *Discussion* are constituted by an interactive type of discourse. *Embedded correction* has a non-conceptual focus as the teacher asks funneling questions without probing deeper to understand the underlying reasons for the error. *Discussion* has a conceptual focus as the teacher involves the students while trying to understand the underlying reasons for the error. Since the unit of analysis is whole episodes of teachers' error-handling practices, categories from previous literature such as questions are part of the interactive episodes coded as *Embedded correction* and *Discussion*. For example, "Hint to the same/other student" (Santagata, 2005) can be parts of *Embedded correction* and asking a "why" question (Santagata, 2005) or "asking for certainty or agreement" (Schleppenbach et al., 2007) can be parts of *Discussion*. Ingram et al. (2015) suggest that the teacher initiates a repair regarding the student's response instead of correcting it (e.g., asking additional, prompting questions to give the student an opportunity to correct themselves). The repair can be part of either *Embedded correction* or *Discussion*, depending on the non-conceptual or conceptual nature of the interactive discourse that unfolds. Regarding *Discussion*, elements of both probing and embracing errors (Brodie, 2013) can be present, which can contribute to developing students' reasoning and raise students' awareness of their errors (Gardee & Brodie, 2015). We discovered quite early on that these four categories did not cover all teachers' error-handling practices. New inductive categories were uncovered and specified through a joint discussion after performing a new review of the episodes of these practices. This is especially the case for the three categories, *Reflect*, *Ignore use later*, and *Praise*, which—as far as we know—are not described in the literature on teachers' error-handling practices. In line with Table 1, a similar overview was made to describe the other four categories in Table 2.

Table 2 The other four categories of the analytical framework, with the aspects of interactivity, foci, and description

	Interactive/non-interactive	Foci
Ignore error	Non-interactive	The error is not dealt with at all
Ignore error use later	Non-interactive first, and then interactive/non-interactive later when the error is brought up	First, the error is not dealt with; later, the error can be handled in either a conceptual or non-conceptual way
Reflect error	Interactive or non-interactive	Conceptual or non-conceptual depending on how students handle the error
Praise error	Non-interactive	The error is (wrongly) praised

Ignore corresponds to Schleppenbach et al.'s (2007) category "ignoring the error" and is framed by Gardee and Brodie (2015) as teachers' absence or avoidance of dealing with a learner error. Different reasons for ignoring student errors might be that the teacher deliberately chooses not to engage with the erroneous response or that the teacher did not hear or notice the response (Gardee & Brodie, 2015). *Reflect*, *Ignore use later,* and *Praise* stem from the data. Further descriptions of these categories will be given below.

In line with Gardee and Brodie (2015), but unlike previous research (i.e., Santagata (2005) and Schleppenbach et al. (2007)), whole sequences (i.e., episodes) of teachers' error-handling practices were coded in the present research instead of only coding the teacher's response immediately following a student's error. There are advantages and disadvantages with these two approaches: coding whole episodes can better capture the nature of teachers' interactive error-handling practices, but at the same time, is more comprehensive for the coder than coding only individual turns since episodes are of different lengths and number of turns. Having clear criteria for the start and end of episodes was thus crucial, which in the present research was defined as such: an episode starts when a student error occurs, an error here defined as a mathematically incorrect statement or answer. An episode ends when either of the following occurs: (a) the error is resolved, (b) the teacher leaves a student/a group of students, (c) another task is undertaken, or (d) another error occurs. The following eight different categories for how teachers handle errors were thus created:

Direct correction

The teacher corrects the error, either by saying the right answer, telling the students s/he is wrong or showing by means of body language that the error is wrong. For example, a student is asked to compare the fractions $\frac{2}{5}$ and $\frac{4}{7}$ and replies that they are same. The teacher says, "No, they are not similar, $\frac{4}{7}$ is bigger than $\frac{2}{5}$."

Embedded correction

The teacher attempts to lead the student with one or more questions or comments towards the right answer. For example, a student is asked how many sixths there are in a whole and replies that there are 12. The teacher follows up with a leading question: "You have a pizza divided into six slices. If you are going to eat the whole pizza, how many slices have you eaten?" The student then answers, "Six."

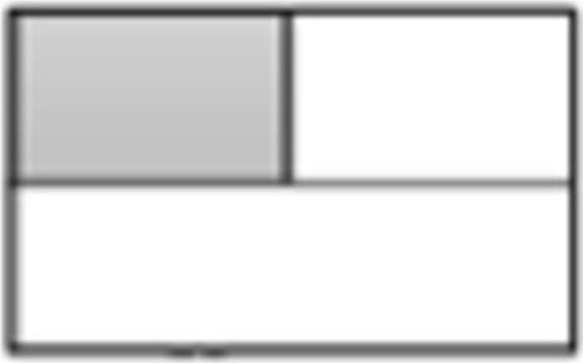
Teacher explanation

A *teacher explanation* of the error is a non-interactive talk when the teacher makes a conceptual explanation in relation to the error made by a student. This means that the teacher emphasizes key mathematical concepts in response to the student error.

For example, a teacher elaborates on the difficulties students have in comparing the size of $\frac{2}{5}$ to $\frac{4}{7}$. Several students have tried to compare them by drawing circles divided into five and seven parts; however, the drawings were too inaccurate. The teacher says: "Sometimes it could be nice to draw circles, but sometimes it could be hard as you have to draw them quite exactly, and in this case, it is hard to draw the circles with the right shape and compare them. It would be easier to compare the fractions if you use a paper with squares and use the squares to draw a rectangle." Then, the teacher draws two rectangles divided into five parts and seven parts, respectively.

Discussion

The teacher invites the student/s to engage in a conceptual discussion about the error, and the students contribute by giving different suggestions and explanations regarding the error. For example, the students are discussing the shaded part of Fig. 1 as a fraction and their suggestions include that it is $\frac{1}{4}$, $\frac{1}{3}$, and $\frac{1}{2}$. The teacher turns to those who have answered $\frac{1}{3}$ and asks them to explain their thinking without correcting this suggestion. Another student does not agree with their explanations and says the answer must be $\frac{1}{4}$: "If you divide the largest part in two, you will get four parts." When the teacher asks why they should draw a new line and divide the figure in four, another student answers that the figure must have equal parts.

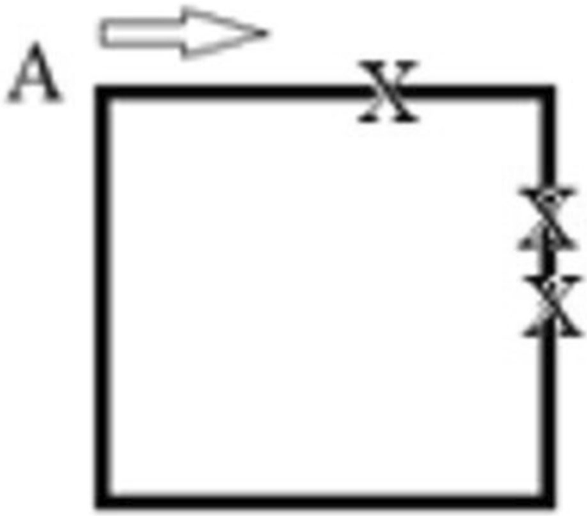


Ignore

By ignoring, the teacher does not pay attention to the error made by the student; the teacher does not say or use embodied language or long pauses to signal that the statement made by a student is incorrect. For example, the teacher does not say anything if a student claims that $\frac{2}{5}$ is greater than $\frac{4}{7}$. In situations where students spoke simultaneously and one student said something incorrect, and it seems that the teacher may have missed noticing what that student said, we did not interpret this an error. Similarly, if a student had written an error in his/her notes but that part of the notes was not in focus of the conversation between the teacher and the student, we did not code such an instance as an error since it is likely that the teacher did not notice the error.

Ignore use later

The way this category differs from the former is that although the error is initially ignored, the teacher registers the error and uses it in a subsequent stage of the lesson. This error-handling practice was thus coded twice. First, it is coded as *Ignore use later* when the teacher ignores the error, and then in the subsequent situation in which the error is emphasized and used by the teacher for further instruction it is coded as another category. For example, the students are working on a task to mark a cross where they think one third of the distance around a square is (refer Fig. 2), and the teacher listens to the students' suggestions without intervening. This instance is coded as *Ignore use later*. At the end of the lesson, the teacher focuses on three different student solutions, marked with a cross in Fig. 2. She draws a square on the board and marks the different solutions in order to ask: 'How could we figure out which of these marks are the right one? What do you think'? and the students contribute with suggestions. This latter instance is coded as *Discussion*.



Reflect

The teacher focuses on an error made by one or several students and asks them to think or talk further about the error. For example, a teacher observes a calculation two students have done and says: "Till here the calculation is right, very good! Then from this line to the next, something's happened. Read through it once more. Think about it once more. You're almost there."

Praise

The teacher praises a mathematically incorrect statement made by a student by saying that it is good. This should not be confused by the teacher praising an aspect of the error and then correcting the error, as this error-handling practice would be categorized as *Direct correction*. Within this category, it can also be the case that the teacher praises other aspects of the error made by a student than just the error itself, i.e., the teacher ignores the mathematical aspect and praises aspects as having a suggestion, making an everyday example etc. For example, students have written an everyday example to 0.6–0.4: A pizza is divided into ten parts, Melissa ate six slices and Anne four slices. The teacher says: "You have made an everyday example for the task. Very good example." The teacher praises the everyday example made by the students even though the example is not correct. Note that this category should not be confused with praising an error as a positive opportunity for learning.

As can be seen from the examples above, errors are predominantly of a conceptual nature (e.g., related to the concept of fractions), which is the case for the data set as a whole due to the conceptually and problem-solving oriented lessons in Boost for Mathematics.

In this section, we explore in more detail the quality of teachers' error-handling practices. In the discussion, we will elaborate on different teacher profiles in terms of the quality for students' learning, and therefore, we will briefly describe how we conceptualize this quality. Regarding the quality of teachers' error-handling practices, we consider some practices as more valuable for students' opportunities to learn. Regarding the interactive error-handling practices, we consider *Discussion* as more valuable than *Embedded correction* since students are involved in a conceptually grounded discussion in the practice *Discussion*. Regarding the non-interactive error-handling practices, we consider *Teacher explanation* as more valuable than *Direct correction*, owing to the fact that the teacher explaining the underlying conceptual reasons for why an error is not correct gives students' richer opportunities to learn than just correcting the error (Brodie, 2014; Radford et al., 2011). Further, *Ignore use later* is considered more valuable than just *Ignore*. The quality of *Ignore use later* is however dependent on how the teacher acts when the error is raised in a subsequent stage of the lesson. *Reflect* implies that the teacher hands over the authority (Ingram et al., 2015) to the student(s) after having pointed out something specific to discuss in relation to an error. This could be an opportunity for students' learning, but the teacher leaves the students alone and hence does not have control of the quality of students' reasoning. Lastly, we consider *Praise* as a low-quality error-handling practice as the teacher praises a student error, and in such situations the students will probably believe that their answer is correct.

Method of analysis

The analysis for the present research was performed in the following manner: The first and second author coded lessons from six teachers each, 26 and 25 lessons respectively. In the process, the first and second author studied their respective lessons several times. Each coder marked all the instances when a student error occurred in a lesson, i.e., the time and the error that occurred. The following episode was then coded according to one of the eight categories in the analytical framework described above. In Excel, one sheet per teacher was created for the coded error episodes for each lesson, as well as including extra descriptors such as time, category, description of the error and the lesson phase. Regarding lesson phase, according to Jackson et al. (2013), a common lesson structure in a reform-oriented middle-grade mathematics curriculum is the three-phase lesson. A complex task is introduced, students work on solving the task, and the teacher orchestrates a concluding whole-class discussion. Heinze (2005) separates private error-handling practices into individual (students are working on their own) and small groups/pairs. Based on the literature, we have decided to divide the lesson into four phases: Introduction of the task, students working alone, students working in pairs or groups and whole-class discussion. Note that the different phases of the lesson were not always structured linearly as some teachers skipped certain phases of the lesson (e.g., letting the students work in groups directly after the introduction of the task), or conducted a shorter whole-class discussion in the middle of the lesson.

We aimed to capture situations wherein the teacher handled a mathematical error in the classroom and investigate the moment when the teacher ignored an undisputable mathematically incorrect statement. Regarding *Ignore* it might either be the case that the teacher deliberately chose to not handle the error or that the teacher did not notice the error. Therefore, it is not as clear with *Ignore* as with the other error-handling practices whether the teacher actually viewed the statement as an error. However, considering that we excluded errors that were only peripherally written in student notes (without pointed to or spoken about) and errors that were uttered by the students talking to one another (in parallel with the teacher focusing on the utterance of another student), it can reasonably be assumed that the errors we coded as *Ignore* were noticed by the teacher, and thus, the teacher chose to deliberately ignore the error.

An inter-rater reliability test was conducted by randomly selecting lessons from each coder (first and second author) to be coded by the other coder using the categories from the analytical framework for teachers' error-handling practices. After each of the coders completed the coding, the marked times and descriptions of student errors were compared, and after reaching an agreement upon which time stamps should be included in relation to the student errors, the respective categorizations of the error-handling were compared. This procedure was repeated two lessons at a time (one lesson from each coder) until 12 lessons had been double-coded and compared: one lesson per teacher of the 12 teachers in the study. Therefore, 12 out of the 51 lessons (24%) were compared between the two coders to assess inter-rater reliability according to: (a) occurred student errors and (b) categorization of episodes. The inter-rater agreement for the time stamps of student errors was found to be 87%, which could be considered a substantial inter-rater reliability (Stemler, 2004). In almost all the cases of disagreement, the reason was that one of the authors had overlooked the error, rather than a disagreement whether the episode should be coded as an error handling episode or not. The inter-rater agreement for the categorization of episodes was found to be 92% with a Cohen's kappa of 0.95, which could also be considered a substantial inter-rater reliability (Landis & Koch, 1977; Stemler, 2004). After the inter-rater reliability test, each of the two coders went through the rest of their respective lessons once more to make sure that any of the latest distinctions that were agreed upon as part of the inter-rater reliability process were not overlooked.

We registered 491 episodes of error-handling practices of teachers. Considering that we focused on handling of conceptual errors of the teachers, we classified errors types by using the categories slips and conceptual errors from Gardee and Brodie (2015). A conceptual error is caused by a misconception (Gardee & Brodie, 2015) or a developing conception of a newly learnt content. The first and second authors coded all 491 instances as either a conceptual error or a slip. Disputes were resolved by discussion until agreement was reached upon whether it was considered a conceptual error or a slip. In total, only 22 (4.5%) instances were considered as slips, with the remaining 469 instances of conceptual errors that we should focus on. An overview of teachers' error handling practices across the whole lesson can be found in appendix. The conceptual errors thus dominate as for Gardee and Brodie (2015) which is not surprising due to the conceptually and problem-solving oriented lessons in the "Boost for Mathematics." The teachers in our study handled the slips by *Direct correction* (10), *Embedded correction* (5), *Reflect* (3), and *Ignore* (4), which is natural because slips are caused by carelessness, and it would not make sense to have a long discussion or offer an explanation in relation to a slip. Regarding lesson phases, no disagreements were found in the process. After the coding was completed, the actions of each teacher in various lesson phases were summarized for all her/his lessons in a separate document. The diagrams depicting an overview of the error-handling practices of each teacher in divergent lesson phases for all her/his lessons were also created (refer Diagram 1). For instance, a summary of one of the teachers' actions in different lesson phases includes the following information (which is illustrated in Diagram 1):

Lisa,[3] **Summary**: *Here, it is clear that the teacher solely uses Direct correction in the introduction of the task. In the other phases, it is more varied, but Direct and Embedded correction dominate individual and pair work. In whole-class discussion, it is more varied between different error-handling practices.*

[3] Pseudonyms are used to ensure the confidentiality of the participants.

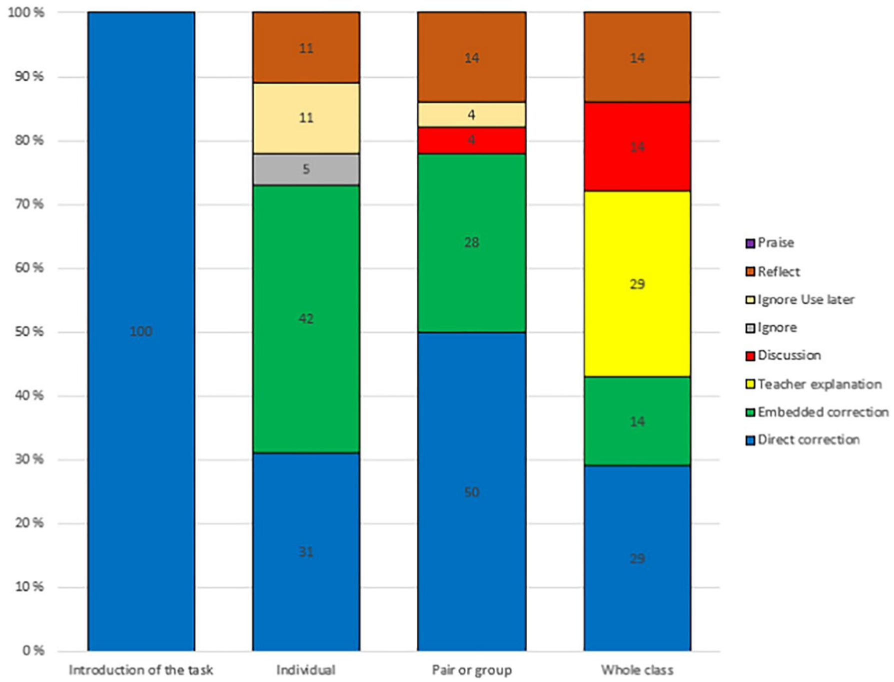


Diagram 1 Summary of Lisa's error-handling practices for all her lessons

A cross-case analysis was conducted to compare the lesson phases for each of the individual teacher cases to capture any existing patterns in teacher responses to errors, i.e., across the lesson phases. The proportion of error-handling practices in different lesson phases (as illustrated for one teacher in Diagram 1) was compared between individual teacher cases. By looking for similarities and differences between teachers' error-handling practices across lesson phases, different error-handling teaching profiles emerged.

Results

The results will be presented in two parts. The first part describes teachers' error handling practices in different phases of the lesson. The second part describes five error-handling teaching profiles we identified across individual teacher cases.

Teachers' error-handling practices in different lesson phases

After analyzing 51 lessons from 12 different teachers, 469 episodes were registered where the teacher handled a conceptual error. The total number of registered errors in each phase of the lesson was 38 in the introduction of the task,

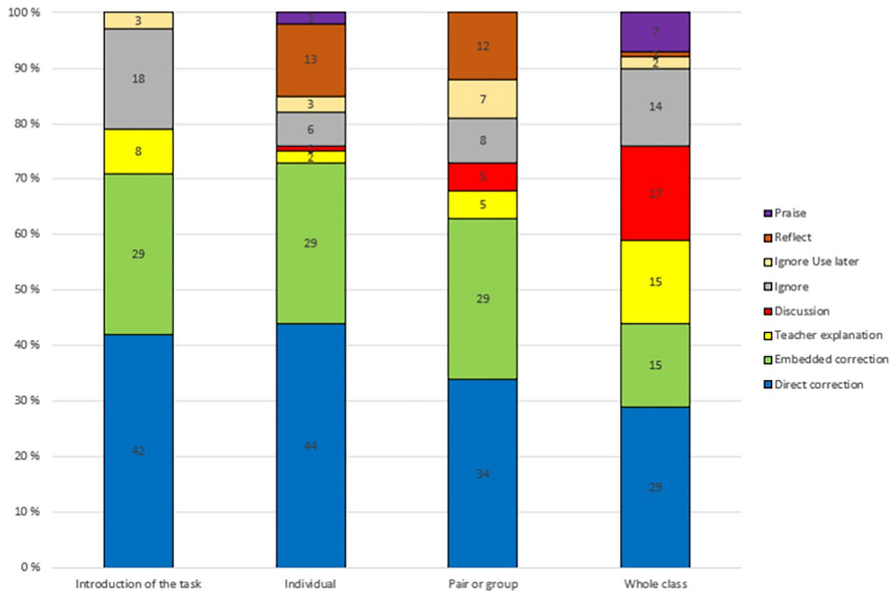


Diagram 2 Teachers' error-handling practices in different phases of the lesson

144 when students worked individually, 184 when they worked in pairs/group, and 103 in whole-class discussions. Diagram 2 summarizes all 12 teachers' error-handling practices as a percentage of the different lesson phases.

The categories of *Direct correction* and *Embedded correction* dominated all four of the lesson phases. In terms of the former, it was by stating the correct answer or saying the answer was wrong either verbally or non-verbally through body language, and in terms of the latter, it was by guiding the students with a series of questions to help the student(s) to state the correct answer. In regard to the category of *Discussion*, this was found to be more likely to occur as part of the whole-class phase, but was also found to be totally absent in the introduction of the task. Although in the introduction and whole-class phases the teachers usually engaged with the whole class, the distribution of *Discussion* was found to differ considerably which will be further elaborated in the discussion section. *Discussion* was also found to occur during individual work and pair/group work, but only to a small extent. Except for the introduction of the task, *Teacher explanation* was found to have a very similar distribution between the phases as the category of *Discussion*. One possible way of explaining these similarities is that the error-handling practices teachers generally use during the introduction of a task tend to minimize or steer students' contributions. From this perspective, avoiding *Discussion* and *Reflect* and instead correcting, ignoring, and explaining makes sense.

Ignore was found to be relatively equally distributed across the phases and was found, surprisingly enough, to be used more frequently compared to *Teacher explanation*. Other studies have found that teachers ignore errors to a large extent (e.g., Gardee & Brodie, 2015; Schleppenbach et al., 2007). *Ignore use later* was found to be most dominant when students were working in pairs or groups. Most often the teacher returned to these errors as part of a whole-class discussion by explaining or discussing the error that had occurred earlier. Although the category of *Praise* was found to seldom occur (only two teachers praised errors repeatedly), when they did occur it was as a part of whole-class discussions to praise an error without correcting it. It is only possible to speculate upon why teachers use such an error-handling practice, but one guess is that the teacher praises students' working process rather than digging into the error. The *Reflect* category was found to be most likely used when students were working alone or in pairs/groups which was not a surprising finding. In this category, when the teacher asked a question or made a statement to the student(s) with the aim of having them reflect further the teacher often moved away to leave the students to reflect by themselves.

Error-handling teaching profiles

To be able to draw conclusions not only about the aggregated proportion of error-handling practices in different lesson phases for all the teachers, but also about individual teachers' error-handling practices across lesson phases, an analysis focusing on the lesson phases for each teacher was conducted. In addition, a cross-case analysis was made across the individual teacher cases to look for similarities and differences between different teachers' error-handling practices across the lesson phases in order to create teaching profiles with similar handling of student errors across the lesson phases. Five error-handling teaching profiles thus emerged among the twelve teachers in the study. Although in the first three teaching profiles the teachers correct students' errors extensively during students' work time, the first three profiles were found to differ in regards to teachers' error-handling practices during the whole-class phase. The following five teaching profiles emerged:

(1) Correcting errors throughout all phases

This profile is characterized by teachers who used corrections extensively—both *Direct corrections* and *Embedded corrections*—in the introduction of the task, in individual and small-group work as well as in the whole-class phase. Two teachers (Tracy[4] and Brenda) mainly adhered to this pattern. For Tracy, *Embedded corrections* dominated, whereas for Brenda, *Direct corrections* dominated. Another teacher (Jennifer[5]) could be partly identified as adhering to this teaching profile as she used *Direct correction* in all the phases and used *Embedded correction* extensively during small-group work. Apart from making corrections, however, Jennifer made use of *Teacher explanations* several times in small-group work and whole-class phase, as well being in the minority of teachers who gave an explanation in the

[4] For Tracy, a third dominating error-handling practice in the introduction of the task was to ignore the error.

[5] In Jennifer's case, the number of errors was low in all the lesson phases except for small-group work.

introduction phase. The baseline for her error-handling practices was thus *Corrections*, even though she used *Teacher Explanations* to a certain extent.

(2) Correcting errors during students' work while few errors are brought up in whole class

Teachers in this profile mainly corrected errors with *Direct and Embedded correction* in individual and small-group work as well as the introduction of the task (even though there were no errors made during Nicole's introduction). However, in contrast to the first teaching profile, very few errors were highlighted by the teacher and the students in the whole-class phase. It might be the case that these teachers (Amy and Nicole), and their students tried to avoid errors in the whole-class phase. Worth mentioning is that both these teachers praised mathematically incorrect statements several times: Amy as part of whole-class work, and Nicole in individual work.

(3) Correcting errors during students' work while using a variety of practices in whole class

Teachers in this profile mainly corrected students' errors with *Direct and Embedded corrections* in individual and pair work. In the whole-class phase, however, the teachers used a variety of different ways of handling errors, e.g., *Teacher explanation, Discussion, Reflect*, *Direct, and Embedded correction* (see Diagram 1 for Lisa's error-handling practices). In other words, the teachers' actions in the whole-class phase differed in significant ways compared to the first two teaching profiles. Two teachers (Lisa and David) adhered to this pattern. Lisa's error-handling practice in the introduction phase was to use *Direct correction*, whereas David's introduction produced no errors.

(4) Ignoring errors while using some of them in whole class

Teachers in this profile were found to mainly *Ignore* errors during individual and pair work but noted and handled some of the previously ignored errors in the whole-class discussion using different error-handling practices. Two teachers (Patricia and Rebecca) adhered to this pattern. However, in individual work Rebecca also performed quite a lot of corrections of students' errors, while Patricia only used *Reflect*. Rebecca deliberately used student errors in one lesson in particular, with a large amount of errors, emphasizing the most frequently occurring errors during the whole-class phase. No student errors were found to occur in the introduction of the task of Patricia and Rebecca's lessons (except for one error that was ignored by Patricia).

(5) Discussing and explaining errors

In the last profile, teachers were found to use *Discussion* and/or *Teacher explanation* extensively in the whole-class phase, digging conceptually into students' errors and misconceptions that occurred. *Discussion* and *Reflect* were also prominent practices in individual and/or small-group work. Two teachers (Melissa and Sarah) acted according to this pattern, which can be seen as a conscious way of handling errors in line with reform ideas. During these teachers' lessons, no student errors were found in the introduction of the task, and consequently, there were no errors to handle during this lesson phase. Another teacher (Eric) can be partly identified as acting in line with this teaching profile, but not adhering to it entirely. Eric conducted entire lessons consisting of either individual and small-group work or whole-class work (apart from the introduction phase). His most often used error-handling practice during small-group work was *Reflect*, and in the whole-class phase, he began with handling errors by *Discussion* in the first half and by making *Direct corrections* in the second half. This shift might be explained due to lack of time as this teacher seemed to be striving to implement the reform ideas for handling student errors.[6] In summary, we identified five distinct error-handling profiles across 12 teacher cases. These error-handling profiles will be further elaborated post discussion on the results in relation to different lesson phases in a broader sense.

Discussion

This article contributes to research about the error-handling practices of teachers. Research has focused on the error handling practices of teachers in different settings and related to factors such as culture (Santagata, 2005; Schleppenbach et al., 2007), teacher beliefs, and knowledge (Bray, 2011) and error types (Gardee & Brodie, 2015). As far as we know, only Santagata (2005) studied error-handling practices in various lesson phases. She also examined an average percentage of mistakes per lesson discussed publicly versus privately in Italy and the USA. We add to this by characterizing the patterns of error-handling practices in different lesson phases. The research question focuses on what characterizes teachers' error-handling practices of students' conceptual errors within and across different lesson phases. The analysis also reveals that teachers handle errors differently in divergent lesson phases. Further, the public and private error-handling practices of teachers also differ, and we will elaborate on these differences.

The analysis of the phases

The results exhibit a great difference between the private and public error-handling practices of teachers, especially between private lesson phases and whole-class discussion. In the private activities of the students working alone and in small groups, three dominating error-handling practices were found, namely, *Direct correction, Embedded correction,* and *Reflect* (refer Diagram 2). Conversely, in the public activity of the whole-class discussion, error-handling practices of teachers, namely, *Direct correction*, *Embedded correction*, *Discussion*,

[6] How different situations such as time pressure influence how teachers handle student errors will be further explored in forthcoming publications.

Teacher explanation, and *Ignore* were more equally distributed than in the private activities.

Direct correction and *Embedded correction* are common error handling practices that occur in all the four phases of the lesson. In addition to these two error handling practices, we find in each phase of the lesson different dominating types of error handling practices. In the introduction of the lesson, *Ignore* is the most common error handling practice beside *Direct correction* and *Embedded correction*. In this phase of the lesson, the majority of teachers do not probe further into errors and students are therefore given few opportunities to clarify and justify their ideas (White, 2003). Approximately one fifth of the errors in the introduction of the lesson are ignored (refer Diagram 2). Other studies, such as Gardee and Brodie's (2015) study, have found that errors are relatively often ignored in classroom settings. It might be different explanations as to whether teachers ignore students' errors in the introduction of the lesson, but it seems that teachers strive towards keeping this phase focused on launching the mathematical problem, rather than dwelling into students' misconceptions. We guess that the teachers both want to keep this phase short and avoid too many potentially confusing mathematical discussions but further studies are needed.

When students are working alone or in pairs, *Reflect* is the most common error handling practice beside *Direct correction* and *Embedded correction*. Reflect implies that the teacher hands over the authority (Ingram et al., 2015) to the student(s) after having pointed out something specific to discuss in relation to an error. The set-up of error-handling practices during these phases is in resonance with the idea that teachers should not disturb students during their mathematical work, hence being somewhat invisible (Valoyes-Chávez, 2019) or non-intervening (cf. Kaufmann & Ryve, 2022), but further studies are needed both for understanding the reasons for and the function of these practices.

A larger variation of error-handling practices was found in a whole-class discussion including *Discussion* and *Teacher explanation*. In a whole-class discussion, commonly at the end of the lesson, the teacher obtains the opportunity to bring up errors that she/he had noticed during individual/group work. In such situations, the teacher thus involves the students in the discussion more than in other phases. How teachers handle students' errors can have significant implications to students' learning (Ingram et al., 2015) and using students' errors as springboards for inquiry has been emphasized by several scholars (Bray, 2011; Brodie, 2014; Gardee & Brodie, 2015, 2021; Ingram et al., 2015; Santagata, 2005; Schleppenbach et al., 2007). It can be argued that when the teacher uses students' errors to set up a discussion with the class, the errors can act as productive pathways for building new concepts (Boaler, 2015). Students can clear up the misunderstanding in order to be able to work further with the problem and are encouraged to talk about the reasoning behind the error (Fennema et al., 1996). Although research has investigated teachers' error-handling practices in whole-class situations (Bray, 2011; Ingram et al., 2015; Schleppenbach et al., 2007), we characterized patterns of error-handling practices across different lesson phases. Our result reveal that the use of *Discussion* and *Teacher explanation* is most prominent in whole-class discussion, but not in the other phases of the lesson.

Teaching profiles

Previous research has affirmed that teachers predominantly correct and ignore student errors (Gardee & Brodie, 2015). In line with the results from previous studies, the error-handling practices of teachers are dominated by correcting or ignoring student errors in the first four teaching profiles that emerged in our study. Nonetheless, there are crucial differences among these four teaching profiles in how teachers handle student errors. By taking a closer look at how the error-handling practices of teachers differ in different lesson phases, the picture from previous studies on error-handling practices is further nuanced. The five error-handling teaching profiles that emerged in our study are new findings in the research literature, and the teaching profiles are here discussed in relation to previous research and implications for learning.

In the first three profiles, teachers mainly correct errors in individual and small-group works. Nevertheless, in the whole-class phase, it differs significantly how teachers act. In the first profile, *Correcting errors through all phases*, errors are mainly corrected, and also in the whole class. In the second profile, *Correcting errors during students' work while few errors are brought up in the whole class*, few errors are highlighted in the whole class. These profiles can be seen in relation to the finding of Bray (2011) that some teachers avoid the discussion of erroneous solutions publicly. Given the lack of student–teacher discussion in relation to students' errors in both these profiles, students have minimal opportunities to talk about their mathematical thinking (Fennema et al., 1996) and to discuss conceptual underpinnings of their errors (Bray, 2011). This way of handling errors prevents errors from constituting a fundamental part of students' learning in the whole-class phase, which is in stark contrast to the Italian teachers in Santagata's (2005) study and the Chinese teachers in Schleppenbach et al.'s (2007) study who publicly discussed a majority of errors. In the third profile, *Correcting errors during students' work while using a variety of practices in whole-class,* errors are used as opportunities for learning through public discussions to a certain extent in the whole-class phase (cf. Bray, 2011).

In the fourth profile, *Ignoring student errors while using some of them in whole class*, a prominent feature is that teachers ignore student errors that they have found during individual and pair work and use some of them later in the whole-class phase. In other words, teachers ignore errors in private, but use errors to a certain extent in public. A large proportion of the errors are however ignored altogether without being followed up in the whole class. The quality of how errors are followed up in the whole class also differs. The intentionality level (cf. Bray, 2011) might hence be high in terms of focusing on student errors in this teaching profile, but this is not always the case. The intentionality level depends on how the teacher uses the errors in the whole class, i.e., to which degree the errors are employed as an opportunity to learn or not. In contrast to Chinese teachers in the study of Schleppenbach et al. (2007), who intentionally used student errors for discussing common errors, the teachers in the fourth profile did not use the errors intentionally, except in some cases.

In the fifth profile, *Discussing and explaining errors*, teachers discuss and explain students' errors extensively in the whole-class phase, and to some extent also when students work alone or in pairs. Errors are publicly–and to some extent privately–handled by digging into conceptually founded reasons for the errors. This profile is related to promoting conceptual understanding through the discussion of errors (cf. Bray, 2011). Such error-handling practices are indicative of a more productive maths teaching style to foster reasoning by asking "why questions" instead of giving corrections (Radford et al., 2011).

In summary, in the fifth error-handling teaching profile errors are used productively to a higher extent, i.e., utilize errors as opportunities for learning (Brodie, 2014; Ingram et al., 2015; Swan, 2006) and as springboards for inquiry (Borasi, 1994; Bray, 2011). The fourth teaching profile has the potential to create opportunities for learning from errors, but how productive it becomes relies on how teachers actually deal with errors when they are brought up in the whole class. Additionally, the third teaching profile to a certain extent adheres to the principle that learning may occur through the public discussions of errors, which is not the case for the first and second teaching profiles in which errors are mainly corrected or avoided in the whole-class setting. Although the findings of this study provide insights into teachers' error-handling practices, it is important to note that this study also has several limitations. First, it has a limited scope. In particular, this study focuses on teachers from the fourth, fifth, and sixth grades. Thus, studying the error-handling practices of teachers in Grades 1–3 and 7–9 could be of interest to future researchers who aim to compare the different grade levels. Second, this study highlights teachers' error-handling practices within and across lesson phases. However, we have not considered the influence of teachers' beliefs and knowledge on their error-handling practices, or whether such practices hold across different settings. In some of the teachers' lessons, numerous errors occurred. The amount of errors committed during a lesson may have affected their error-handling practices.

It is the authors' hope that designers of professional development programs in mathematics education might benefit from the findings of the present study. A significant challenge in accomplishing ambitious reform is the way teachers view their students' capabilities (Jackson et al., 2017), which can further be revealed by how teachers handle student errors in the classroom. The findings of this study reveal how teachers handle errors differently in divergent lesson phases. Therefore, teachers must know these differences to create springboards for inquiry (Borasi, 1994; Bray, 2011) by giving students time and opportunities to discuss their errors with their peers in small groups and in the whole-class setting. Further research is essential to investigate potential reasons for these five error-handling teaching profiles. In some lesson sequences, especially in individual and pair works, we observed fairly considerable disturbance and time pressure. The teacher needed to handle many questions and errors. Further research could also consider whether—and how—the error-handling practices of teachers are affected by these factors. Moreover, by connecting teacher beliefs and mathematical knowledge for teaching to the error-handling practices of teachers, we answer questions as (1) under what circumstances teachers decide and manage to handle students' errors in productive and conceptually founded ways and (2) what might be the reasons for such teachers to correct and ignore student errors instead.

Appendix

Table 3

Table 3 Overview of teachers' error handling practices

	Introduction of the task								Individual								Pair or group								Whole class							
	Dc	Ec	Te	Di	Ig	Iu	Re	Pr	Dc	Ec	Te	Di	Ig	Iu	Re	Pr	Dc	Ec	Te	Di	Ig	Iu	Re	Pr	Dc	Ec	Te	Di	Ig	Iu	Re	Pr
Amy	4	4							15	11	2		1				6	9			1		2					1				7
Jennifer	2		1						2								12	12	6	1			2		4	1	2		1			
Lisa	5								6	8			1	2	2		14	8		1		1	4		2	1	2	1			1	
Melissa																		1	1	3	3		2					6	2			
Nicole									7	4			2		5	3	6	9			2		2						1			
Patricia	1														5		1	1				6	2		1	1			1	2		
David									10	2		1	1												1		2	1	1			
Rebecca									13	8			2	3	4						5	6	2		6	3	4	1	2			
Eric										1	1				1		1						3		4		1	4	1			
Tracy	4	7	2		7	1				2							3	6			2		2		10	8	2	1	4			
Brenda									10	5			2		2		19	7	2		2		2		2				1			
Sarah																				4			1			1	2	3				

Dc Direct correction, *Ec* Embedded correction, *Te* Teacher explanation, *Di* Discussion, *Ig* Ignore, *Iu* Ignore use later, *Re* Reflect, *Pr* Praise

Funding Open Access funding provided by Ostfold University College

Open Access This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article's Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit http://creativecommons.org/licenses/by/4.0/.

References

Boaler, J. (2015). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages and innovative teaching*. John Wiley & Sons.

Borasi, R. (1994). Capitalizing on errors as 'springboards for inquiry' – A teaching experiment. *Journal for Research in Mathematics Education, 25*(2), 166–208. https://doi.org/10.2307/749507

Bray, W. S. (2011). A collective case study of the influence of teachers' beliefs and knowledge on error-handling practices during class discussion of mathematics. *Journal for Research in Mathematics Education, 42*(1), 2–38. https://doi.org/10.5951/jresematheduc.42.1.0002

Brodie, K. (2013). The power of professional learning communities. *Education as Change, 17*(1), 5–18. https://doi.org/10.1080/16823206.2013.773929

Brodie, K. (2014). Learning about learner errors in professional learning communities. *Educational Studies in Mathematics, 85*(2), 221–239. https://doi.org/10.1007/s10649-013-9507-1

Bryman, A. (2016). *Social research methods*. Oxford University Press.

Fennema, E., Carpenter, T. P., Franke, M. L., Levi, L., Jacobs, V. R., & Empson, S. B. (1996). A longitudinal study of learning to use children's thinking in mathematics instruction. *Journal for Research in Mathematics Education*, *27*(4), 403–434. https://doi.org/10.2307/749875

Gardee, A., & Brodie, K. (2021). Relationships between teachers' interactions with learner errors and learners' mathematical identities. *International Journal of Science and Mathematics Education*, *20*(1), 193–214. https://doi.org/10.1007/s10763-020-10142-1

Gardee, A., & Brodie, K. (2015). A teacher's engagement with learner errors in her Grade 9 mathematics classroom. *Pythagoras, 36*(2), 293. https://doi.org/10.4102/pythagoras.v36i2.293

Heinze, A. (2005). Mistake-handling activities in the mathematics classroom. In H. L. Chick & J. L. Vincent (Eds.), *Proceedings of the 29th conference of the international group for the psychology of mathematics education* (Vol. 3, pp. 105–112).

Ingram, J., Pitt, A., & Baldry, F. (2015). Handling errors as they arise in whole-class interactions. *Research in Mathematics Education, 17*(3), 183–197. https://doi.org/10.1080/14794802.2015.1098562

Jackson, K., Garrison, A., Wilson, J., Gibbons, L., & Shahan, E. (2013). Exploring relationships between setting up complex tasks and opportunities to learn in concluding whole-class discussions in middle-grades mathematics instruction. *Journal for Research in Mathematics Education, 44*(4), 646–682. https://doi.org/10.5951/jresematheduc.44.4.0646

Jackson, K. J., Gibbons, L., & Sharpe, C. J. (2017). Teachers' views of students' mathematical capabilities: Challenges and possibilities for ambitious reform. *Teachers College Record, 119*(7), 1–43. https://doi.org/10.1177/016146811711900708

Lampert, M., Beasley, H., Ghousseini, H., Kazemi, E., & Franke, M. L. (2010). Using designed instructional activities to enable novices to manage ambitious mathematics teaching. In M. K. Stein & L. Kucan (Eds.), *Instructional explanations in the disciplines* (pp. 129–141). Springer. https://doi.org/10.1007/978-1-4419-0594-9_9

Landis, J. R., & Koch, G. G. (1977). The measurement of observer agreement for categorical data. *Biometrics, 33*(1), 159–174.

National Council of Teachers of Mathematics [NCTM] (2000). *Principles and standards for school mathematics*. Author.

Radford, J., Blatchford, P., & Webster, R. (2011). Opening up and closing down: How teachers and TAs manage turn-taking, topic and repair in mathematics lessons. *Learning and Instruction, 21*(5), 625–635. https://doi.org/10.1016/j.learninstruc.2011.01.004

Santagata, R. (2005). Practices and beliefs in mistake-handling activities: A video study of Italian and US mathematics lessons. *Teaching and Teacher Education, 21*(5), 491–508. https://doi.org/10.1016/j.tate.2005.03.004

Scheiner, T. (2020). Dealing with opposing theoretical perspectives: Knowledge in structures or knowledge in pieces? *Educational Studies in Mathematics, 104*(1), 127–145. https://doi.org/10.1007/s10649-020-09950-7

Schleppenbach, M., Flevares, L. M., Sims, L. M., & Perry, M. (2007). Teachers' responses to student mistakes in Chinese and U.S. mathematics classroom. *The Elementary School Journal, 108(2),* 131–147. https://doi.org/10.1086/525551

Skolverket (2018). *Matematiklyftet [Boost for Mathematics].* Retrieved from https://larportalen.skolverket.se/#/moduler/1-matematik/alla/alla

Stemler, S. E. (2004). A comparison of consensus, consistency, and measurement approaches to estimating interrater reliability. *Practical Assessment, Research, and Evaluation, 9*, 4. https://doi.org/10.7275/96jp-xz07

Swan, M. (2006). Designing and using research instruments to describe the beliefs and practices of mathematics teachers. *Research in Education, 75*(1), 58–70. https://doi.org/10.7227/RIE.75.5

Valoyes-Chávez, L. (2019). On the making of a new mathematics teacher: Professional development, subjectivation, and resistance to change. *Educational Studies in Mathematics, 100*(2), 177–191. https://doi.org/10.1007/s10649-018-9869-5

White, D. Y. (2003). Promoting productive mathematical classroom discourse with diverse students. *The Journal of Mathematical Behavior, 22*(1), 37–53. https://doi.org/10.1016/S0732-3123(03)00003-8

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.