Digital Fabrication for Mathematics Education: A Critical Review of the Field

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This article presents a critical review on digital fabrication for creating manipulatives in mathematics education research. It provides an overview of the research field from two perspectives: 1) explored digital fabrication technologies and reified mathematical concepts, and 2) applied pedagogical theories and research methodologies. Results show that 3D printing is the most common technology for creating manipulatives reifying mathematical concepts within geometry, algebra, and functions, as well as fractions. In the typical research project, a qualitative paradigm is chosen, and learning is investigated from a constructionistic perspective. The review reveals a small but trending research community and concludes with two specific opportunities for consolidating research on digital fabrication for mathematics education.

Keywords: Critical review, digital fabrication, manipulatives, mathematics education.

Introduction

This article presents a critical review of previous research on digital fabrication for mathematics education. Digital fabrication (DF) is “the process of translating a digital design developed on a computer into a physical object” (Berry et al., 2010, p. 168). DF technologies such as 3D-printers, laser cutters or vinyl cutters, have become affordable and can be found at Makerspaces and FabLabs[1] around the world. These tools enable people to create professionally looking items rapidly and at a relatively low cost. Blickstein (2013) argues that DF and making can play a major role in education, “bringing powerful ideas, literacies, and expressive tools to children” (p. 2). DF technologies might support mathematics teachers through the creation of manipulatives. Manipulatives or concrete material are physical objects that are used to reify abstract concepts in mathematics education. Previous research highlights, that students benefit from long-term use of manipulatives in mathematics education. Both students’ achievements in and motivation for mathematics increase when manipulatives are used (Pires et al., 2019). However, manipulatives are not self-explanatory and to draw maximum benefit from students’ use of manipulatives, teachers must continuously situate their activities and the physical objects based on students’ previous experiences, as well as their teaching context (Thompson, 1992). Stylianou (2010) has found that teachers have knowledge gaps of how different representations, such as manipulatives, are translated into mathematical concepts. DF technologies could enable teachers to create context-sensitive manipulatives for teaching activities, situated in the classroom and enhance teachers’ knowledge on manipulatives to improve their teaching. There is prior research (Ford & Minshall, 2019; Hielscher & Smith, 2014; Papavlasopoulou et al., 2017) reviewing DF education in general. To my knowledge, there does not exist a review focusing on mathematics education and how DF can be used to create manipulatives,

[1] https://fabfoundation.org/global-community/

which is the objective in this article. I performed a critical review of previous research in the field to answer the following research question: *What characterizes research on digital fabrication for creating manipulatives in mathematics education?*

Method

The aim of this review is to explore how DF has been used in mathematics education research so far, focusing on DF technologies as well as mathematical concepts and manipulatives on one hand and used theories and research methodologies on the other. I applied a three-step iterative critical analysis (Çorlu et al., 2017; Stigberg, 2017): finding appropriate papers including selection of databases and search terms; eliminating irrelevant papers based on a set of exclusion criteria; analyzing appropriate papers based on key terms answering the research questions. Grant and Booth (2009) describe that "a critical review goes beyond mere description of identified articles and includes a degree of analysis and conceptual innovation" (p. 93). In this review, I will analyze identified articles, highlight research gaps and provide opportunities for further research in DF for mathematics education.

Step 1: Retrieving Publications

The database used as primary source for finding literature is Ebsco, including Education Source, Education Research Complete, Academic Search Primier, ERIC, CINAHL, MathSCINet via EBSCOhost and MEDLINE. To broaden the search and ensure to include as many relevant publications as possible, I chose to perform a search on Google Scholar as well.

From the research topic and research questions, I derived the following keywords: "digital fabrication", "3D-printing" or "laser-cutting", in combination with "mathematics education" and "manipulatives" or "concrete materials". 3D-printing and laser-cutting are popular DF technologies and often used as representatives or synonyms of DF and therefore highly relevant as keywords. A search for vinyl-cutter, another DF technologies gave no hits and was excluded from the final search query. In mathematics curricula, both terms manipulatives and concrete material are used to describe physical objects that reify a mathematical concept and were included in the search. Finally, I used '*' special character in the search query to allow different forms of all keywords. I did not define a timeframe since I was interested in finding all relevant literature. The final queries used in the search can be found at: shorturl.at/eqxP4. The search queries were used for a full text search in both Ebsco and Google Scholar. The literature search resulted in 73 hits from Ebsco and 277 hits from Google Scholar. After removing duplicates, the total number of found articles is 254.

Step 2: Appropriate papers

In this step, the papers that were not appropriate for the review were eliminated based on the following exclusion criteria: not peer-reviewed (36), non-English work (10), not empirical (purely theoretical papers or review articles) (66), do not concern mathematics education or manipulatives not investigated (159), studies on STEM projects not specifically highlight mathematics education or creating manipulatives (81), or 3D-pen as technology (5). Several publications corresponded to more than one exclusion criteria. For a complete list of inclusion/exclusion criteria, see link: shorturl.at/aioyU. The elimination process resulted in a final list of 17 papers to be included in the analysis. The included literature can be found in **Table *1***.

Table 1: Overview of included articles

Code	Article
A	Paul, S. (2018). 3D Printed Manipulatives in a Multivariable Calculus Classroom. Primus: Problems, *Resources & Issues in Mathematics Undergraduate Studies, 28*(9), 821–834. https://doi.org/10.1080/10511970.2018.1445675
B	Wan, Anna and Jessica Ivy. "Adding a New Dimension to Teaching Mathematics Educators." *Handbook of Research on TPACK in the Digital Age*, edited by Margaret L. Niess, et al., IGI Global, 2019, pp. 390-412. https://doi.org/10.4018/978-1-5225-7001-1.ch018
C	Ulbrich, E., Lieban, D., Lavicza, Z., Vagova, R., Handl, J., & Andjic, and B. (2020). *Come to STEAM. We have cookies!* 297–304.
D	Fernández, E., Davidson, J., & Pomponio, E. (2021). *Dare to Care: The Impacts of a Caring Pedagogy on Mathematical Making, Teaching, and Learning*.
E	Junthong, N., Netpradit, S., & Boonlue, S. (2018). *Design and Development of Teaching Tools in Dimensional Geometry for Visually Impaired Students Using Object Models from 3D Printing*. 7. https://doi.org/10.17758/HEAIG2.H0418464
F	Greenstein et al. (2020). Exploring the interwoven discourses associated with learning to teach mathematics in a making context Greenstein, S., Jeannotte, D., Fernández, E., Davidson, J., Pomponio, E., & Akuom, D. (2020). Exploring the interwoven discourses associated with learning to teach mathematics in a making context. *Conference Papers Psychology of Mathematics & Education of North America*, 840–844. https:/doi.org/10.51272/pmena.42.2020.
G	Corum, K., & Garofalo, J. (2016). Learning about Surface Area through a Digital Fabrication-Augmented Unit. *Journal of Computers in Mathematics and Science Teaching*, *35*(1), 33–59.
H	Greenstein, S., & Seventko, J. (2017). *Mathematical Making in Teacher Preparation: What Knowledge Is Brought to Bear?* North American Chapter of the International Group for the Psychology of Mathematics Education.
I	Akuom, D., & Greenstein, S. (2021). *Prospective Teachers' Design Decisions, Rationales, and Resources: Re/claiming Teacher Agency Through Mathematical Making*.
J	Greenstein, S., & Olmanson, J. (2017). Reconceptualizing Pedagogical and Curricular Knowledge Development Through Making. *Design Journal*, *4*, 10.
K	Greenstein, S., Fernández, E., & Davidson, J. (2019). Revealing teacher knowledge through making: A case study of two prospective mathematics teachers. *Conference Papers -- Psychology of Mathematics & Education of North America*, 1151–1156.
L	Hallowell, D. A. (2020). *Spatial Reasoning in Elementary School Children's Geometry Insight: A Neo-Piagetian Developmental Proposal*.
M	Mohamed, M. M., Paoletti, T., Vishnubhotla, M., Greenstein, S., & Lim, S. S. (2020). Supporting students' meanings for quadratics: Integrating RME, quantitative reasoning and designing for abstraction. *Mathematics Education Across Cultures: Proceedings of the 42nd Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, 193–201. https:/doi.org/10.51272/pmena.42.2020

N Ceragioli, F., & Spreafico, M. L. (2020). Tangible Tools in Mathematics for Engineering Students: Experimental Activity at Politecnico di Torino. *Digital Experiences in Mathematics Education*, *6*(2), 244–256. https://doi.org/10.1007/s40751-020-00063-7

O Davidson, J., Fernández, E., & Greenstein, S. (2019). Teachers making manipulatives to promote pedagogical change. *Conference Papers -- Psychology of Mathematics & Education of North America*, 1359–1360. eue.

P Dilling, F., & Witzke, I. (2020). The Use of 3D-Printing Technology in Calculus Education: Concept Formation Processes of the Concept of Derivative with Printed Graphs of Functions. *Digital Experiences in Mathematics Education*, *6*(3), 320–339. https://doi.org/10.1007/s40751-020-00062-8

Q Corum, K., & Garofalo, J. (2016). Learning about Surface Area through a Digital Fabrication-Augmented Unit. *Journal of Computers in Mathematics and Science Teaching*, *35*(1), 33–59.

Step 3: Analyzing the papers

Quantitative and qualitative data was extracted from the articles and gathered in a database. The analysis of each paper included general bibliographic information, as well as type of DF technologies, reified mathematical concepts, applied theories, research design and methodology.

Results

There is limited research on DF technologies for creating manipulatives in mathematics education (17). Most of the initially found publications (254) present research on STEM (81) or do not focus mathematical content or manipulatives (159). Often research presents interdisciplinary projects to facilitate students design thinking and innovation skills. Furthermore, searching on Google Scholar gave several articles that were not peer reviewed (36) such as teaching material or blog entries. Research on 3D pens was excluded based on the definition of DF as computer-controlled tools (Gershenfeld, 2012). Most of the research on DF technologies included in this review is published 2018 or later (14) indicating a growing interest in DF technologies for education. Furthermore, the research community around DF in mathematics education is limited to a small number of research groups. For example, Greenstein published 7 out of the 17 articles (F, H, I, J, K, M, O). A major research project on DF technologies in mathematics education is a project concerning pre-service teachers called: Prospective Teachers Making for Mathematical Learning (F, I, K). In the following, I have analysed the 17 articles from two perspectives: Firstly, focusing on DF technologies and created artefacts reifying mathematical concepts. Secondly, focusing on applied theories, research design and methodologies.

Digital fabrication technologies

3D printers are the most used DF technology for creating manipulatives (15). Two articles, based on the same project, explore the use of a die cutter as DF technology (G, Q). Four articles provide a rationale for their choice of DF technology. 3D printers are starting to become cheap and accessible (A, B), enable teachers and students to create artifacts with little effort and in reasonable time (C, P) and the possibility to reproduce existing artefacts (P). The reviewed articles present different software choices for modelling 3D objects: TinkerCad (C, J, N), OpenScad (C, P), SketchUp (E) and AutoDesk 123D Design (L). FabLab Model Maker was used to design 3D models for the die cutter technology (G, Q). Nine articles do not provide information about a specific 3D modelling software. Wan and

Ivy (2019) mention Thingiverse[2], an online platform where makers can share their work, as one possible resource for finding 3D models for mathematics education.

Artifacts reifying mathematical concepts

The reviewed articles present three core mathematical concepts: geometry, algebra and functions, and fractions. Geometrical objects such as rectangular prisms and cubes (C, D, G, Q), cones (J, N), prisms representing triangles (M), tessellation with pattern blocks (D), general geometric properties (L) and 3D printed geoboards for representing area and volume (E). Created objects reifying concepts of algebra and functions such as: coordinate system (J), 3D printed representations of graphs of functions with one variable (P) and two variables where students investigate contours, partial derivates, gradient vector field, and restrictions to the curve (A), and models of the integral as area under a graph (J). Mohamed et al. (2020) presents different 3D printed triangular prism to investigate students' reasoning about quadratic changes. 3D printed objects reifying fraction as value (F, I) and fraction of time using circle segments (K, O).

Opportunity 1: Exploring alternative technologies, added mathematical concepts, and resources for creating and sharing manipulatives

The results expose that previous research on DF in mathematics education is sparse. There is a need to explore different DF technologies, alternative manipulatives reifying more mathematical concepts, and investigating the use of available online resources. Most of the research has been done using 3D printers, limiting the type of manipulatives that can be created through additive manufacturing. Laser cutter or vinyl cutter enable subtractive manufacturing and open a new design space for manipulatives. So far, laser cutter technologies are expensive, but one could argue that they will become more commonplace in schools like 3D printers today. Manipulatives are created for geometry, algebra, and functions, as well as fractions. One interesting research path is investigating how manipulatives can reify other concepts such as arithmetic, decimal system, combinatorics, or probability, all are part of the mathematics curriculum. Research has focused on how students and teachers can use software to create 3D models that can be printed. There are plenty of online resources available for teachers to choose from e.g., Thingiverse. Understanding how manipulatives can be shared by teachers, taking part in the maker culture is missing. From prior professional development projects, I often met teachers requested pre-made material or lesson plans they can adopt to their own teaching. They express a lack of time to develop their own material on the one hand, but they prefer to be creative and develop their own teaching materials on the other hand.

Applied theories

Three theoretical characteristics have emerged: 1) constructivism and constructionism as theoretical underpinnings, 2) design theories for creating a learning context, and 3) theories for analysing teachers' and students' knowledge. Firstly, many of the reviewed articles refer to constructivism and constructionism as rationale when creating manipulatives (9). Papert's constructionism (Papert & Harel, 1991) is based on Piaget's constructivism (Piaget, 2013), but emphasizes that learning is

[2] https://thingiverse.com

happening when we create sharable things (Ackermann, 2001). Secondly, the research group around Greenstein frames the process of creating manipulatives as learning by design (H,I,K). The approach presented by Koehler and Mishra (2005) provides an "opportunity to consider the interplay between the evolving artifact and the application of teacher knowledge domains in the artifact's development" (Greenstein et al., 2019, p. 1152). Thirdly, teachers' technological, pedagogical, and content knowledge (TPACK) is explored in three articles (B,H,K). Dilling and Witzke (2020) apply theory of subjective domain of experience when analysing students' knowledge. According to this theoretical framework, students experience is linked to the specific learning context and their knowledge needs to be described according to their situational link, including previous experiences. Greenstein et al. (2020) is an article that investigates learning from a sociocultural perspective, using theory of commognition (Sfard, 2007) to analyse students' changes in discourses. In their article, they specifically analyse, students' narratives about; mathematical objects, participants of the discourse, learning about mathematics, and design decisions. An overview can be found at: shorturl.at/mnsN3.

Research methodologies

The included articles present different cases how DF was used in a teaching context. Most commonly, researchers report on their own teaching experiences in higher education (10), on interventions applying manipulatives in a classroom setting for K-12 students (5), or from a workshop or course for in-service teachers (2). Most articles (14) use purely qualitative methods in their research design based on observation, video recordings, interviews, and hand-ins. Three articles (B,H,L) use mixed methods to collect data including surveys, and pre- and post-tests. Research on teaching experiences in higher education analyse cases based on selected students and their work (D,F,I,J,K,M,O), data from students' self-reports (B,H), or description of the course and used manipulatives (A,N). Research projects involving K-12 students investigate interventions with manipulatives developed by researchers in a classroom setting (E,L), and students' learning outcomes when creating manipulatives (G,P,Q). Ulbrich et al. (2020) report on their experiences from a workshop series with 200 in-service teachers. They conclude that it is essential to learn more "about a teacher's needs and expectations using technologies" (Ulbrich et al. 2020, p. 303). Greenstein & Olmanson, (2017) provide a case of a DF course for in-service teachers including examples of created artefacts. Seven articles did not use a framework for analysing their results.

Opportunity 2: Adopting communities of practice as a framework for understanding in-service teachers learning of digital fabrication for mathematics education

The typical research project investigates DF for pre-service teachers applying constructionism (Papert & Harel, 1991) as theoretically underpinning describing learning from an individual perspective. Constructionism provides an acknowledged framework for understanding learning when creating and sharing manipulatives. However, for professional development projects with in-service teachers, a sociocultural perspective on learning is preferable, because it offers a "more effective means to, understand and implement an educational partnership for work-place learning" (Spouse, 2001, p. 512). The review found two research projects exploring how in-service teachers could use DF to create their own manipulatives and only one research project applies a sociocultural perspective. To consolidate research on DF for mathematics education both perspectives (individual and sociocultural) need to be investigated appropriately for both pre-and in-service teachers.

Communities of practice (CoP) deploys a situated learning perspective with sociocultural elements and is widely accepted in education research and work-place learning (Hammersley, 2005). CoP could provide a theoretical lens on how learning emerges when pre- and in-service teachers engage in authentic learning experiences, such as making, sharing, and using available manipulatives to develop their professional identity in their community (Wenger, 1999). CoP has been proposed as a suitable research paradigm for the mixed methods approach, which has been found in three articles included in this review (Denscombe, 2008).

Conclusion

This paper presents a critical review of 17 research articles on DF for creating manipulatives in mathematics education. Previous research is concentrated at stray research clusters. The typical DF research project explores manipulatives for reified mathematical concepts in geometry, algebra and functions, and fractions, using 3D printing and is published 2018 or later. Qualitative methods or mixed methods are predominant for data collection. The review reveals three main theoretical perspectives. Firstly, constructionism, with roots in constructivism (Papert & Harel, 1991), as underpinning theories. Secondly, design theories, such as Learning by Design (Koehler & Mishra, 2005), and thirdly, theories for analysing teachers' and students' knowledge. Finally, the paper provides two specific opportunities for consolidating research on DF for mathematics education: 1) Exploring alternative technologies, added mathematical concepts, and resources for creating and sharing manipulatives, and 2) Adopting communities of practice as a framework for understanding in-service teachers learning of DF for mathematics education.

References

Ackermann, E. (2001). Piaget's constructivism, Papert's constructionism: What's the difference. *Future of Learning Group Publication*, *5*(3), 438.

Berry, R. Q., Bull, G., Browning, C., Thomas, C. D., Starkweather, G., & Aylor, J. (2010). Use of digital fabrication to incorporate engineering design principles in elementary mathematics education. *Contemporary Issues in Technology and Teacher Education*, *10*(2), 167–172.

Çorlu, D., Taşel, Ş., Turan, S. G., Gatos, A., & Yantaç, A. E. (2017). Involving Autistics in User Experience Studies: A Critical Review. *Proceedings of the 2017 Conference on Designing Interactive Systems*, 43–55. https://doi.org/10.1145/3064663.3064771

Corum, K., & Garofalo, J. (2015). Using digital fabrication to support student learning. *3D Printing and Additive Manufacturing*, *2*(2), 50–55. https://doi.org/10.1089/3dp.2015.0008

Denscombe, M. (2008). Communities of Practice: A Research Paradigm for the Mixed Methods Approach. *Journal of Mixed Methods Research*, *2*(3), 270–283. https://doi.org/10.1177/1558689808316807

Ford, S., & Minshall, T. (2019). Invited review article: Where and how 3D printing is used in teaching and education. *Additive Manufacturing*, *25*, 131–150. https://doi.org/10.1016/j.addma.2018.10.028

Gershenfeld, N. (2012). How to make almost anything: The digital fabrication revolution. *Foreign Aff.*, *91*, 43.

Grant, M. J., & Booth, A. (2009). A typology of reviews: An analysis of 14 review types and associated methodologies. *Health Information & Libraries Journal*, *26*(2), 91–108. https://doi.org/10.1111/j.1471-1842.2009.00848.x

Hammersley, M. (2005). What can the literature on communities of practice tell us about educational research? Reflections on some recent proposals1. *International Journal of Research & Method in Education*, *28*(1), 5–21. https://doi.org/10.1080/01406720500036653

Hielscher, S., & Smith, A. (2014). *Community-Based Digital Fabrication Workshops: A Review of the Research Literature.* (SSRN Scholarly Paper ID 2742121). Social Science Research Network. http://dx.doi.org/10.2139/ssrn.2742121

Koehler, M. J., & Mishra, P. (2005). Teachers Learning Technology by Design. *Journal of Computing in Teacher Education*, *21*(3), 94–102. https://doi.org/10.1080/10402454.2005.10784518

Mayer, R. E. (2004). Should There Be a Three-Strikes Rule Against Pure Discovery Learning? *American Psychologist*, *59*(1), 14–19. https://doi.org/10.1037/0003-066X.59.1.14

Papavlasopoulou, S., Giannakos, M. N., & Jaccheri, L. (2017). Empirical studies on the Maker Movement, a promising approach to learning: A literature review. *Entertainment Computing*, *18*, 57–78. https://doi.org/10.1016/j.entcom.2016.09.002

Papert, S., & Harel, I. (1991). Situating constructionism. *Constructionism*, *36*(2), 1–11.

Piaget, J. (2013). *Child's Conception of Space: Selected Works vol 4* (Vol. 4). Routledge.

Sfard, A. (2007). When the Rules of Discourse Change, but Nobody Tells You: Making Sense of Mathematics Learning from a Commognitive Standpoint. *Journal of the Learning Sciences*, *16*(4), 565–613. https://doi.org/10.1080/10508400701525253

Spouse, J. (2001). Bridging theory and practice in the supervisory relationship: A sociocultural perspective. *Journal of Advanced Nursing*, *33*(4), 512–522. https://doi.org/10.1046/j.1365-2648.2001.01683.x

Stigberg, S. K. (2017). A Critical Review on Participation in Mobile Interaction Design Research. *MUM 2017: The 16th International Conference on Mobile and Ubiquitous Multimedia, November 26--29, 2017, Stuttgart, Germany*, 161–166. https://doi.org/10.1145/3152832.3156629

Tseng, F.-C., & Kuo, F.-Y. (2014). A study of social participation and knowledge sharing in the teachers' online professional community of practice. *Computers & Education*, *72*, 37–47. https://doi.org/10.1016/j.compedu.2013.10.005

Wenger, E. (1999). *Communities of practice: Learning, meaning, and identity*. Cambridge university press