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Equity induced up-trading and the housing market structure: Implications for Price-to-Income (PTI) and macroprudential interventions

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Abstract:
This paper relates the housing market structure and the intensity of equity induced up-trading between market segments to the equilibrium ratio between house prices and household income (PTI-ratio). When fed back into the housing market through up-trading, the equity gains from home ownership that accompany appreciations might induce short-run overreactions in house prices compared to developments in household income. The aggregate equity gain available to households trading up the housing ladder, and the corresponding overreaction, is contingent on the housing market structure. The paper derives the housing market structure necessary to eliminate short run overreactions and discuss which structures that carries with them the strongest overreactions. Also, by taking the indirect effects arising from equity induced up-trading into account we relate market structure and equity to two distinct applications: (i) The short run development of the price-to-income (PTI) ratio. (ii) The price effects of macroprudential interventions.

Keywords: Housing market structure, equity, up-trading, PTI.

1. Introduction

For most households the life cycle adaption to the housing market involves trading up a ladder. As a young adult most of us live in small rented dwellings. As we age, marry and have kids, we seek larger dwellings, and to a greater extent owner-occupied housing. The market segments for owner-occupied housing can thus be described as a step-wise relation between a starter, an intermediate and a family home segment (See for instance Ortalo-Magne' and Rady (2006)). The ability to trade up the ladder is often related to the equity gains from the existing housing stock (Kajuth, 2010).

In addition to allow individual households to adjust their housing consumption, equity gains might impact on market developments. For instance, Borgersen and Sommervoll (2011) show how equity gains bring about short-run deviations in the price-to-rent (PR) ratio. Larsen (2010) argues the equity gains from home ownership to be an important source of housing market instability, while equity gains in Borgersen (2012) amplify shocks and introduce a multiplier into the housing market

When analyzing house price developments in Europe between 1985 and 2007 Hilbers et al (2008) distinguishes between fast lane, average performers and slow movers. Despite a common positive momentum, the paper finds variations in house price growth across Europe. For instance, with the exception of the slow movers, house prices started appreciating much faster than income in the beginning of this century (Hilbers et al, 2008, p. 14). These deviations contrast the often argued long run relation between house prices and household income (see for instance Capozza et al, 2002).

This paper extends the model of Borgersen and Sommervoll (2011) and relates short-run deviations in the price-to-income (PTI) ratio to the prevailing housing market structure and the intensity of equity induced up trading between segments. The purpose is twofold:

First, we consider how a homogenous housing market model that ignores the interplay between segments leaves out an important shifter in the housing market: That is the role of equity.

To highlight the role of equity and the interplay between segments we consider two scenarios: First, a shock to demand equal across market segments. We show how equity results in short-run overreactions in the PTI-ratio. Second, a shock to demand constrained to *one* of the market segments. To give this latter scenario some purchase we relate it to macroprudential interventions and consider a shock to the demand for starter homes, leaving the other demand shifters unaffected. A macroprudential intervention like a cap on LTV-ratios can be argued to have significant effects on the ability of first-time entrants to enter owner occupation, but - impacting on only a part of the housing market - to be without any substantial effect on house prices. Still, empirical results indicate that prices are affected (see for instance, Crowe (2011)). We show how such interventions might result in non-negligible price effects when both direct and indirect effects are accounted for.

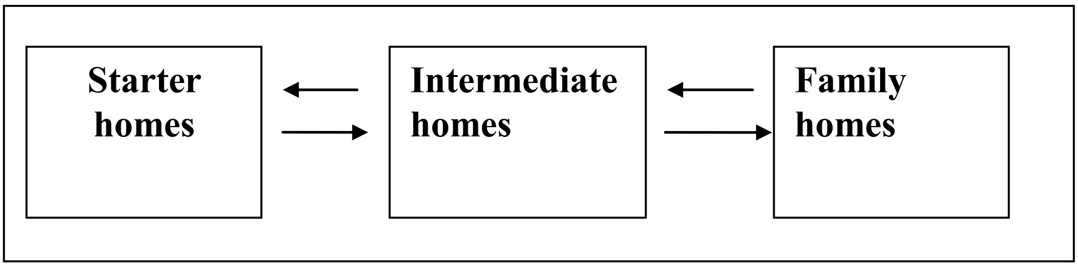
Second, to highlight the context specific nature of the equity induced short run overreactions we integrate a heterogeneous housing market structure with housing market equity and discuss which structures that are are most likely to induce overreactions in the equilibrium indices. The paper shows that it is *not only* the strength of the equity induced up-trading, but also its distribution between segments that matter for short run overreactions in the PTI-ratio.

The paper is structured as follows: The heterogeneous housing market model of Borgersen and Sommervoll (2011) is given in Section 2, where we also distinguish between our stylized housing market regimes. Section 3 relates the housing market structures that ensure constant PTI- ratios to the distribution of equity induced up-trading between segments across our four regimes. The last part concludes.

2. Equity gains and a heterogeneous housing market structure

The linear model of Borgersen and Sommervoll (2011) introduces a housing market with three segments for owner-occupied housing; starter (s)-, intermediate (m)- and family (f) homes. When abstracting away from the rental market, the presumed housing ladder can be illustrated as in Figure 1.

Figure 1: The housing market ladder for owner-occupied housing



Market segment prices ensure equilibrium in each segment. To highlight the short-run nature of the model supply is fixed in all segments and the equilibrium condition equals:

1) $$D_i = S_i \qquad i = s,m,f$$

where, S_i, is housing supply and, D_i, is demand in market segment i. The price index for the housing market as a whole is:

2) $$P = \Sigma_i \ \alpha_i P_i$$

where market segment sizes, α_i, determine the different segments weights in the house price index.

A household's demand for owner-occupied housing is related to the price of housing, *P,* household's net equity, *E*, and household income, *k*. To simplify, we abstract away from the substitution effect.[1]

In each market segment is demand conditional on the aggregate equity held by households operating on the demand side of this segment. This aggregate equity depends on the price of their existing homes. For households demanding intermediate homes equity is related to the price of starter homes: $E_m = E_m(P_s)$. The same applies to the market of family homes, where equity is related to the price of intermediate homes: $E_f = E_f(P_m)$. We consider linearized versions of these equity functions:

3) $$e_m E_m = e_{ms} P_s$$

4) $$e_f E_f = e_{fm} P_m$$

Where, e_i, represents the equity elasticity of owner-occupation in market segment *i,* and, e_{ms} (e_{fm}), measures how strongly housing equity of starter (intermediate) homes impact on the up-trading to intermediate (family) homes. E_{0m} and E_{0f} are exogenous equity components and where E_s is an exogenous variable. The demand for owner-occupied housing can be expressed as

5) $$D_s = k_s + e_s E_s - p_s P_s$$

6) $$D_m = k_m + e_{ms} P_s - p_m P_m$$

7) $$D_f = k_f + e_{fm} P_m - p_f P_f$$

When inserting for demand and combining the market segment equilibriums in (1) with the house price index (2) we can express the latter as

8) $$P = \frac{\alpha_f}{p_f}[A_f] + \left[\frac{\alpha_m}{p_m} + \frac{\alpha_f}{p_f}\left[\frac{e_{fm}}{p_m}\right]\right][A_m] + \left[\frac{\alpha_s}{p_s} + \frac{\alpha_m}{p_m}\left[\frac{e_{ms}}{p_s}\right] + \frac{\alpha_f}{p_f}\left[\frac{e_{fm}}{p_m}\frac{e_{ms}}{p_s}\right]\right][A_s].$$

(See the appendix for the definition of variables as well as the expressions for both market segment prices and the house price index).

Market segment prices have both direct and indirect effects on the house price index, where the latter is related to equity gains from home ownership. The direct effect $\frac{\alpha_i}{p_i}$ is determined by the sector size and the demand elasticity of the respective market segment. The factors impacting the indirect effects differ between segments, where the family homes segment - being the final step on the housing ladder with no further steps to climb - does not carry with it any indirect effect to the house price index. Indirect effects however are present both in the case of a shock to the demand for starter homes, A_s, and in the case of a shock to the demand for intermediate homes, A_m. The indirect effects arising from the former

[1] See Borgersen and Sommervoll (2011) for a model where the demand side includes substitution. When households are assumed to substitute down the ladder, the effect of substitution is parallel to that of equity induced up-trading.

$\left[\frac{\alpha_m}{p_m}\left[\frac{e_{ms}}{p_s}\right]+\frac{\alpha_f}{p_f}\left[\frac{e_{fm}}{p_m}\frac{e_{ms}}{p_s}\right]\right]$ as well as from the latter type of shock $\left[\frac{\alpha_f}{p_f}\left[\frac{e_{fm}}{p_m}\right]\right]$ are carried through to the house price index alongside the direct effects as equity gains are applied to trade up the housing ladder.

To highlight the relation between the housing market structure and the intensity of equity induced up-trading between segments we introduce four housing market regimes, separated by the type of shocks and the elasticity of demand.

We introduce two type of shocks: One where shocks are equal across market segments, $dA_i = d\overline{A} \; \forall \; \text{i}$. The other is a shock restricted to the starter home segment, i.e. $dA_s > 0, dA_i = 0 \;\; \text{i} \neq \text{s}$.

Likewise, we consider two distinct market structures: One suppressing all differences between the elasticity of demand, $p_s = p_m = p_f = 1$, and one without any restrictions on the relation between elasticities, i.e. $p_s > 0, p_i > 0, \;\; \text{i} \# \text{s}$. Table 1 summarizes the four regimes:

Table 1: Housing market regimes

	$dA_i = d\overline{A} \; \forall \; \text{i}$	$dA_s > 0, dA_i = 0 \;\; \text{i} \neq \text{s}.$
$p_s = p_m = p_f = 1$	**Homogenous housing market regime**	**Homogenous structure exposed to heterogeneous shocks**
$p_s > 0, p_i > 0, \;\; \text{i} \# \text{s}.$	**Heterogeneous structure exposed to homogenous shocks**	**Heterogeneous housing market regime**

A *homogenous housing market regime* is one where all segments are exposed to equal shocks and the price elasticity is equal across segments. The only difference between a true homogenous housing market model and our homogenous housing market regime is that the latter incorporates equity induced up-trading and the resulting interplay between segments. When structures are equal but shocks are allowed to differ between segments the *housing market structure is homogenous but exposed to heterogeneous shocks.* This regime is applied to discuss the price effects of macroprudential interventions. To address the price effects of macroprudential interventions we consider a shock to the demand for starter homes leaving the other shifters unaffected. A *heterogeneous housing market regime* is one where both shocks and elasticities differ between segments. Finally, when shocks are equal but elasticities differ between segments we have a *heterogeneous* *housing market structure exposed to homogeneous shocks.* It is in the latter two scenarios we derive conditions for the housing market structure that is necessary to ensure constant PTI-ratios.

3. Shocks to demand and the housing market regimes

In this section we consider the price effects of shocks to demand across our presumed housing market regimes. We start out by addressing the interplay between segments that a true homogenous housing market leaves out, driven by equity induced up-trading. We consider the short-run deviation in the PTI-ratio (section 3a) and the price effects of macro prudential interventions (section 3b). Then we allow the price responses to differ between segments and consider the housing market structure necessary to eliminate any short-run deviation in the PTI-ratio when shocks are segment specific (section 3c) or equal across segments (section 3d).

3a. A homogenous housing market regime: The PTI-ratio

In a homogenous housing market regime where both shocks to demand and the demand elasticity is equal across segments, we mirror the implications that would come about in a true homogenous housing market model. The only distinction is that this structure incorporates the interplay between segments.

For considering how shocks to income equal across market segments impact the PTI-ratio, we apply the partial derivative of the house price index (8), using the fact that $p_s = p_m = p_f = 1$ and $dA_i = d\overline{A} \ \forall$ i. Knowing that, $\alpha_s + \alpha_m + \alpha_f = 1$, this derivative reduces to

9) $$\frac{dP}{d\overline{A}} = 1 + \alpha_m e_{ms} + \alpha_f e_{fm}\left(1 + e_{ms}\right).$$

We apply the constancy of the PTI-ratio, $\frac{dP}{dA} = 1$, as the long run equilibrium ratio between prices and income. The short run PTI-overshoot can then be expressed as, $\left(\frac{dP}{d\overline{A}} - 1\right)$.

From (9) we easily find the conditions for, $\frac{dP}{d\overline{A}} = 1$, to equal, $e_{ms} = 0$ and $e_{fm} = 0$. Stated differently, in the presence of equity induced up-trading there will be short run deviations in the PTI-ratio.[2]

When allowing for equity induced up-trading between starter and intermediate homes as well as between intermediate and family homes, i.e. $e_{ms} > 0, e_{fm} > 0$, (9) shows how the housing market structure and the

[2] If we abstract away from equity induced up-trading between intermediate and family homes, but allow for equity induced up trading from starter to intermediate homes, $e_{ms} > 0, e_{fm} = 0$, the PTI-ratio reduces to $\frac{dP}{d\overline{A}} = 1 + \alpha_m e_{ms}$. For a constant PTI-ratio, $\alpha_m = 0$ is a necessary condition. The housing market must now only consist starter and family homes, without any equity link between the two. A reversed scenario, that is $e_{fm} > 0, e_{ms} = 0$, gives $\frac{dP}{d\overline{A}} = 1 + \alpha_f e_{fm}$. The necessary condition for no short run deviation in the PTI-ratio is $\alpha_f = 0$ making it necessary to 'cut off' the final step of the housing ladder.

intensity of equity induced up trading between the different segments impact the short run PTI-overshoot (See the appendix for details).

First, the short run PTI-overshoot is positively related both to the size of the intermediate home as well as to the size of the family home segment (*ceteris paribus)*. A market segment's impact on the extent of overshooting is weaker the further up the ladder the segment is located. Often the housing market structure mirrors the population structure of an economy. When the population is dominated by young adults the housing market is dominated by the market segments on the first steps of the ladder, while an elderly population most often is accompanied by a housing market dominated by the final steps of the housing ladder. In these situations the short run deviation in the PTI-ratio will be more pronounced in economies dominated by young adults.

Second, the degree of equity induced up-trading between starter and intermediate homes stimulates the short run PTI overshoot, as does equity induced up-trading from intermediate to family homes, especially in housing markets with a large family home segment. If the size of the intermediate home segment exceeds a critical limit, equity induced up-trading from starter to intermediate homes will have a stronger impact on the short run PTI-overshoot than up-trading between intermediate and family homes. In economies where young adults face tighter credit constraints, and equity induced up-trading is more pronounced between the first steps of the housing ladder, the short run PTI-overshoot will be stronger.

Finally, let us consider the market structure necessary for eliminating any short run overreaction in the PTI ratio in the presence of equity induced up-trading. Inserting for $\frac{dP}{dA} = 1$ in expression (9) - and using the fact that $\alpha_s + \alpha_m + \alpha_f = 1$- makes us able to express the market structure necessary for no short run PTI overshoot, in terms of the starter home segment, α_s^c, as:

10) $$\alpha_s^c = 1 - \alpha_m(1 - e_{ms}) - \alpha_f(1 - e_{fm}(1 + e_{ms}))$$

From (10) we see that α_s^c is increasing both in the extent of equity induced up-trading between starter and intermediate as well as between intermediate and family homes (*ceteris paribus)*. When $e_{ms} < 1$ we find a negative relation between the segments for starter and intermediate homes, while the relation is positive if $e_{ms} > 1$. When $e_{ms} = 1$ is the size of the starter home segment necessary to eliminate any short run deviation in the PTI-ratio unaffected by the size of the intermediate home segment.

When considering the relation between starter and family homes, we see that it is negative when, $e_{fm} < \frac{1}{e_{ms}+1}$ and positive when, $e_{fm} > \frac{1}{e_{ms}+1}$. The size of the starter home segment that is necessary for no short run overreactions in the PTI-ratio is unaffected by the family home segment when, $e_{fm} = \frac{1}{e_{ms}+1}$.

Basically, our reasoning shows how the housing market structure that will eliminate short run overreactions in the PTI-ratio is contingent on the relation between the intensity of equity induced up-trading between the market segments and is therefore context specific.

3b. A homogenous structure and heterogeneous shocks: Macroprudential interventions

We now consider the price effect of a shock to the demand for starter homes while keeping the other shifters unaffected. From the partial derivative of (8), when assuming $p_s = p_m = p_f = 1$ and $dA_s > 0, dA_i = 0$ i $\neq$ s, and using $\alpha_s + \alpha_m + \alpha_f = 1$ the price response equals:

11) $$\frac{dP}{dA_s} = \alpha_s + \alpha_m e_{ms} + \alpha_f e_{ms} e_{fm}.$$

First, in the absence of equity induced up-trading, that is, $e_{ms} = 0$ and $e_{fm} = 0$, (11) reduces to

12) $$\frac{dP}{dA_s} = \alpha_s \qquad \text{where} \qquad 0 < \frac{dP}{dA_s} < 1$$

Expression (12) formalizes the market segment's size effect described in (2). A shock to the demand for starter homes impacts the house price index in a ratio determined by the segment's market size. In the absence of equity induced up trading a shock to the demand for starter homes is not completely passed through to the house price index.

However, when introducing equity induced up-trading the price effects might be substantial. For instance, if $e_{ms} = 1$ and $e_{fm} = 1$ a shock to the demand for starter homes is completely passed through to the house price index.[3]

To highlight the relation between the housing market structure and the intensity of equity induced up-trading, we first consider the condition for the price effect of a shock to the demand for starter homes to equal its size effect, that is $\frac{dP}{dA_s} = \alpha_s$. When inserting for the size effect in (11) we find the market structure

[3] To elaborate on the relation between market structure and the intensity of equity induced up-trading we could allow for equity induced up trading between starter and intermediate homes, but abstract away from that between intermediate and family homes, i.e. $e_{ms} > 0, e_{fm} = 0$. Expression (11) now reduces to, $\frac{dP}{dA_s} = \alpha_s + \alpha_m e_{ms}$. A shock to the demand for starter homes still has a stronger impact on the house price index than what is given by the size effect. The extent of overreaction is determined by the size of the intermediate home segment and the intensity of equity induced up-trading between starter and intermediate homes.

necessary for the price response to equal the size effect as, $\alpha_s = 1 - \alpha_f(1 - e_{fm})$. The starter home segment has to be scaled down by the size of the family home segment and the intensity of equity induced up-trading between the segments for intermediate and family homes. The necessary size of the starter home segment is unambiguously increasing in latter, while it is decreasing (increasing) in the former if $e_{fm} < 1$ $(e_{fm} > 1)$.

Second, we consider the market structure necessary to ensure constancy of the PTI-ratio even when shocks to demand are constrained to the starter home segment, that is to find the conditions for $\frac{dP}{dA_s} = 1$. We find the necessary market structure to equal, $\frac{\alpha_m}{\alpha_f} = \frac{e_{ms}e_{fm} - 1}{1 - e_{ms}}$. If $e_{ms} < 1$, and $e_{fm} > \frac{1}{e_{ms}}$, this condition implies $\alpha_m > \alpha_f$. For $e_{ms} > 1$, and $e_{fm} < \frac{1}{e_{ms}}$, the condition implies, $\alpha_m < \alpha_f$. While the family home segment has to exceed that of intermediate homes when, $e_{ms} < 1$, the relation is reversed when, $e_{ms} > 1$.

The conditions for no short run deviation in the PTI-ratio and the condition for the price effect of a shock to the demand for starter homes are both context specific and related to the intensity of equity induced up-trading between segments.[4]

3c. A heterogeneous housing market regime: Macroprudential interventions

When considering a heterogeneous housing market structure we find the partial derivative of the house price index (8) with respect to the demand for starter homes as:

12) $$\frac{dP}{dA_s} = \frac{1}{p_s}\left[1 + \frac{\alpha_m}{p_m}\left[e_{ms} - p_m\right] + \frac{\alpha_f}{p_f}\left[\frac{e_{fm}e_{ms}}{p_m} - p_f\right]\right]$$

The impact of a shock to demand for starter homes on the house price index is positively related both to the size of the intermediate and to the size of the family home segment, as well as to the intensity of equity induced up-trading between market segments.

Let us then consider the necessary condition for, $\frac{dP}{dA_s} = \frac{\alpha_s}{p_s}$, i.e. that the price effect of a shock to the demand for starter homes equals the market segment's structurally adjusted size effect. Combining this condition with (12) gives

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[4] When restricting equity induced up-trading to the first part of the housing ladder, i.e. $e_{ms} > 0, e_{fm} = 0$, we find the market structure necessary for no short-run overreactions in the PTI-ratio as, $\frac{\alpha_m}{\alpha_f} = \frac{1}{e_{ms} - 1}$. When $e_{ms} = 0$, this condition reduces to $\alpha_m = \alpha_f$, while for $e_{ms} > 1$ the condition implies $\alpha_m > \alpha_f$.

13) $$\alpha_s = 1 - \alpha_m \left[1 - \frac{e_{ms}}{p_m} \right] - \alpha_f \left[1 - \frac{e_{ms} e_{fm}}{p_m p_f} \right].$$

The housing market structure necessary for the price effect of a shock to the demand for starter homes to equal its market segment's size effect is again contingent on the intensity of equity induced up-trading between segments. Table 2 presents four scenarios.

In scenario A where $e_{ms} = 0$ and $e_{fm} = 0$ the necessary size of the starter home segment is equal to its actual market size as given by (2). Moving to scenario B, where we allow for, $e_{fm} > 0$, but keeps, $e_{ms} = 0$, we find the size of the starter home segment to equal that of scenario A.

Table 2: The size on the starter home segment and equity induced up-trading between segments

Scenario	Necessary size on starter home segment	Equity Distribution
A	$\alpha_s^A = 1 - \alpha_m - \alpha_f$	$e_{ms} = 0$ and $e_{fm} = 0$
B	$\alpha_s^B = 1 - \alpha_m - \alpha_f$	$e_{fm} > 0$ and $e_{ms} = 0$
C	$\alpha_s^C = 1 - \alpha_m - \alpha_f + \alpha_m \frac{e_{ms}}{p_m}$	$e_{ms} > 0$ and $e_{fm} = 0$
D	$\alpha_s^D = 1 - \alpha_m - \alpha_f + \alpha_m \frac{e_{ms}}{p_m} + \alpha_f \frac{e_{ms} e_{fm}}{p_m p_f}$	$e_{ms} > 0$ and $e_{fm} > 0$

In scenario C we picture the reversed scenario, i.e. $e_{ms} > 0$ and $e_{fm} = 0$. The necessary size of the starter home segment in scenario C exceeds that of scenario A (and B). Finally, as we in scenario D allow for equity induced up-trading between both segments, i.e. $e_{ms} > 0$ and $e_{fm} > 0$, the starter home segment increases further and exceeds than of scenario C.

When considering the housing market structure that will give an effect on the house price index equal to the starter home segment's structurally adjusted size effect we see how equity induced up-trading between different market segments matter. When up-trading only takes place between intermediate and family homes equity will not impact the housing market structure as $\alpha_s^A = \alpha_s^B$. When, on the other hand, there is up-trading between starter and intermediate homes the market structure is affected, as $\alpha_s^B < \alpha_s^C = \alpha_s^B + \frac{e_{ms}}{p_m}$. If all market segments are linked through equity induced up-trading the necessary size of the starter home segment increases even further, as $\alpha_s^C < \alpha_s^D = \alpha_s^C + \frac{e_{ms} e_{fm}}{p_m p_f}$.

For a shock to the demand for starter homes to only impact the house price index according to the segment's structurally adjusted size effect the housing market must consist of a larger starter home segment. This in order to constrain the indirect effect by reducing its cumulative effects as it is transmitted through the other steps on the housing ladder.

3d. Heterogeneous structures and homogenous shocks: The PTI-ratio

In a heterogeneous housing market where shocks are equal across segments the short-run PTI ratio equals

14) $$\frac{dP}{d\overline{A}}=\frac{\alpha_f}{p_f}+\left[\frac{\alpha_m}{p_m}+\frac{\alpha_f}{p_f}\left[\frac{e_{fm}}{p_m}\right]\right]+\left[\frac{\alpha_s}{p_s}+\frac{\alpha_m}{p_m}\left[\frac{e_{ms}}{p_s}\right]+\frac{\alpha_f}{p_f}\left[\frac{e_{fm}}{p_m}\frac{e_{ms}}{p_s}\right]\right]$$

The short run PTI-ratio is again affected both by the direct as well as by the indirect effects and in the presence of equity induced up-trading there will be short-run deviations in the PTI-ratio.[5] The overshoot is positively related both to size of the segments on top of the ladder and the intensity of equity induced up-trading.

When inserting for, $\frac{dP}{d\overline{A}}=1$, and using $\alpha_s+\alpha_m+\alpha_f=1$, we find the market structure that eliminates the short-run deviations in the PTI-ratio as:

15) $$\alpha_s=p_s\left[1-\frac{\alpha_m}{p_m}(1+e_{ms})-\frac{\alpha_f}{p_f}\left(1+\frac{e_{fm}}{p_m}+\frac{e_{fm}e_{ms}}{p_m p_s}\right)\right]$$

There are again two distinct features impacting the size of the starter home segment: First, the direct effect from the structurally adjusted size effects, $p_s, \frac{\alpha_m}{p_m}, \frac{\alpha_f}{p_f}$. Second, the indirect effect due to equity induced up-trading. While the family home segment does not carry with it any indirect effect, both equity induced up trading between starter and intermediate homes, $\frac{\alpha_m}{p_m}e_{ms}$, as well as between intermediate and family homes, $\frac{\alpha_f}{p_f}\left(\frac{e_{fm}}{p_m}+\frac{e_{fm}e_{ms}}{p_m p_s}\right)$, impact the house price index indirectly. The indirect effect now reduces the size of the starter home segment that is necessary to ensure constant PTI-ratios. Table 3 presents the necessary market structures in four different scenarios.

[5] Take notice of that the condition $\frac{dP}{d\overline{A}}=1$ is stricter than a mere aggregation of the individual structurally adjusted size effects, $\frac{dP}{dA_s}=\frac{\alpha_s}{p_s}$, $\frac{dP}{dA_m}=\frac{\alpha_m}{p_m}$ and $\frac{dP}{dA_s}=\frac{\alpha_f}{p_f}$. When defining constancy of the PTI-ratio as $\frac{dP}{dA}=1$ we simultaneously impose restrictions on the elasticity of different market segments, restrictions which are not needed when the market structure is homogeneous. A sufficient set of conditions for no short run deviation in the PTI-ratio is for instance, $\alpha_i=p_i \forall i$, fixing all structurally adjusted size effects equal to unity.

T*able 3: The size on the starter home segment, equity induced up-trading and constant PTI-ratios*

Scenario	Necessary size on starter home segment	Equity Distribution
A	$\alpha_s^A = p_s\left[1 - \frac{\alpha_m}{p_m} + \frac{\alpha_f}{p_f}\right]$	$e_{ms} = 0$ and $e_{fm} = 0$
B	$\alpha_s^B = p_s\left[1 - \frac{\alpha_m}{p_m} - \frac{\alpha_f}{p_f}\left(1 + \frac{e_{fm}}{p_m}\right)\right]$	$e_{fm} > 0$ and $e_{ms} = 0$
C	$\alpha_s^C = p_s\left[1 - \frac{\alpha_m}{p_m}(1 + e_{ms}) - \frac{\alpha_f}{p_f}\right]$	$e_{ms} > 0$ and $e_{fm} = 0$
D	$\alpha_s^D = p_s\left[1 - \frac{\alpha_m}{p_m}(1 + e_{ms}) - \frac{\alpha_f}{p_f}\left(1 + \frac{e_{fm}}{p_m} + \frac{e_{fm}e_{ms}}{p_m p_s}\right)\right]$	$e_{ms} > 0$ and $e_{fm} > 0$

In the absence of equity induced up-trading, the size on the starter home segment α_s^A is scaled down by the other segment's structurally adjusted size effects. Introducing equity induced up-trading between intermediate and family homes in scenario B, we see how the size of the necessary starter home segment is reduced, $\alpha_s^A > \alpha_s^B$. Likewise, allowing for equity induced up-trading between starter and intermediate homes also reduces the starter home segment compared to that of scenario A, $\alpha_s^A > \alpha_s^C$. Whether α_s^B exceeds α_s^C depends on the intensity of equity induced up-trading among the different market segments. In scenario D, where equity induced up trading is present between all market segments the size on the starter home segment is reduced even further, $\alpha_s^D < \alpha_s^C - \left[\frac{\alpha_f}{p_f}\left(\frac{e_{fm}}{p_m} + \frac{e_{fm}e_{ms}}{p_m p_s}\right)\right]$.
When considering the housing market structure necessary to eliminate short-run deviations in the PTI-ratio we see how the starter home segment is negatively related to equity induced up-trading between segments.

4. Conclusions

Appreciations create equity gains for homeowners. If fed back into the housing market through up-trading, these gains will impact house prices and create short-term deviations in equilibrium indices for the housing market.

This paper relates the extent of overreaction in the PTI-ratio to the housing market structure and the intensity of equity induced up-trading between different segments on the housing ladder. The paper shows how the extent of overreaction is context specific and related to the housing market structure. We derive conditions for the relation between market structure and equity induced up trading that eliminates short-run deviations in the PTI-ratio. The conditions are expressed in terms of the size on the starter home segment, which is related to the distribution of equity induced up-trading between segments. We highlight the context specific nature of the housing market structure that eliminates short-run PTI overshooting. Second, we discuss the price effects of macroprudential interventions. Such interventions are often argued to only directly impact on households entering owner-occupation for the first time and the first steps of the housing ladder with only modest implications for prices. We show how the price effects of such interventions might be non-negligible when both the direct and the indirect effects are taken into account.

A heterogeneous housing market model brings value added to the understanding of housing markets by highlighting the interplay between segments. The indirect effects arising from equity induced up-trading might induce short run overreactions in the PTI-ratio, especially in economies where the population is dominated by young adults and the housing market by starter homes. These indirect effects might also be substantial when credit constraints are tighter towards young adults.

In housing markets where households trade up a ladder in their life cycle adjustment the market structure necessary to eliminate overreactions in the PTI-ratio is contingent on the intensity of equity induced up-trading between segments. As the intensity varies over time so will the market structure necessary for eliminating short-run overreactions in the PTI-ratio. As market structures are slow to respond long lasting deviations must be expected.

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Appendix

Market segment prices and the house price index

By inserting for the equity functions in (3) and (4) into (5) - (7) housing demand in each market segment can be expressed as:

A5) $$D_s = k_s + e_s E_s - p_s P_s$$

A6) $$D_m = k_m + e_{m0} E_{0m} + e_{ms} P_s - p_m P_m$$

A7) $$D_f = k_f + e_{f0} E_{0f} + e_{fm} P_m - p_f P_f$$

where E_s is an exogenous variable. Combined with the equilibrium condition in (1), market segment prices can be derived as:

A8) $$P_s = \frac{1}{p_s}\left[c_s + e_s E_S - S_s\right],$$

A9) $$P_m = \frac{1}{p_m}\left[c_m - S_m + \frac{e_{ms}}{p_s}\left[e_s E_s - S_s\right]\right],$$

A10) $$P_f = \frac{1}{p_f}\left[c_f - S_f + \frac{e_{fm}}{p_m}\left[c_m - S_m + \frac{e_{ms}}{p_s}\left[c_s - S_s\right]\right]\right],$$

where $c_s = k_s$, $c_m = k_m + e_{m0}E_{0m} + \frac{e_{ms}}{p_s}c_s$ and $c_f = k_f + e_{f0}E_{0f} + \frac{e_{fm}}{p_m}c_m$.

The house price index equals:

A11) $$P = \frac{\alpha_f}{p_f}\left[A_f\right] + \left[\frac{\alpha_m}{p_m} + \frac{\alpha_f}{p_f}\left[\frac{e_{fm}}{p_m}\right]\right]\left[A_m\right] + \left[\frac{\alpha_s}{p_s} + \frac{\alpha_m}{p_m}\left[\frac{e_{ms}}{p_s}\right] + \frac{\alpha_f}{p_f}\left[\frac{e_{fm}}{p_m}\frac{e_{ms}}{p_s}\right]\right]\left[A_s\right]$$

where $A_i = c_i - S_i$.

3a. A homogenous housing market regime - Comparative statics

We have the partial derivative as given by expression (9) as

A12) $$\frac{dP}{d\overline{A}} = 1 + \alpha_m e_{ms} + \alpha_f e_{fm}\left(1 + e_{ms}\right)$$

When considering comparative statics on the short run PTI-overshoot $\left[O \equiv \left(\frac{dP}{d\overline{A}} - 1\right)\right]$ we find:

A13) $$\frac{dO}{d\alpha_m} = e_{ms} > 0$$

A14) $$\frac{dO}{d\alpha_f} = e_{fm}(1 + e_{ms}) > 0$$

A15) $$\frac{dO}{de_{ms}} = \alpha_m + \alpha_f e_{fm} > 0$$

A16) $$\frac{dO}{de_{fm}} = (1 + e_{ms})\alpha_f > 0$$

We find the condition for $\frac{dO}{d\alpha_m} > \frac{dO}{d\alpha_f}$ by considering the inequality, $e_{ms} > e_{fm}(1+e_{ms})$. This reduces to $e_{fm} < 1+e_{ms}$.

Likewise, the condition $\frac{dO}{de_{ms}} > \frac{dO}{de_{fm}}$ equals $\alpha_m + \alpha_f e_{fm} > (1+e_{ms})\alpha_f$ and can be rewritten as $\alpha_m > \alpha_f + \alpha_f(e_{fm} - e_{ms})$.

To find the market structure that eliminates short run overreactions in house prices compared to developments in income, we apply (A12) and insert the constancy of the PTI-ratio, $\frac{dP}{d\overline{A}} = 1$. When using the condition, $\alpha_s + \alpha_m + \alpha_f = 1$, and rearranging, the market structure necessary for constant PTI-ratios can be expressed as $\alpha_s^c = 1 - \alpha_m(1-e_{ms}) - \alpha_f(1 - e_{fm}(1+e_{ms}))$.

3b. A homogenous housing market and heterogeneous shocks - Comparative statics

We have the partial derivative as given by expression (11) as

A17) $$\frac{dP}{dA_s} = \alpha_s + \alpha_m e_{ms} + \alpha_f e_{fm} e_{ms}$$

When $e_{ms} > 0, e_{fm} > 0$, the condition $\frac{dP}{dA_s} = \alpha_s$ reduces (A17) to $0 = \alpha_m + \alpha_f e_{fm} e_{ms}$. Inserting for $\alpha_m = 1 - \alpha_s - \alpha_f$, and rearranging gives the necessary market structure as $\alpha_s = 1 - \alpha_f(1-e_{ms})$.

When inserting for the condition $\frac{dP}{dA_s} = 1$, and using the fact that $1 - \alpha_s = \alpha_m + \alpha_f$, the market structure that ensures complete pass-through is found from the expression $0 = \alpha_m(e_{ms} - 1) + \alpha_f(e_{fm}e_{ms} - 1)$. After some rearranging, this reduces to the ratio between intermediate and family homes as given by, $\frac{\alpha_m}{\alpha_f} = \frac{e_{ms}e_{fm} - 1}{1 - e_{ms}}$.

3c. A heterogeneous housing market regime – Comparative statics

When both price elasticities and shocks differ between segments the partial derivative of (8) with respect to increased demand for starter homes equals

A18) $$\frac{dP}{dA_s} = \left[\frac{\alpha_s}{p_s} + \frac{\alpha_m}{p_m}\left[\frac{e_{ms}}{p_s}\right] + \frac{\alpha_f}{p_f}\left[\frac{e_{fm}}{p_m}\frac{e_{ms}}{p_s}\right]\right]$$

When inserting for $\alpha_s = 1 - \alpha_m - \alpha_f$ this can be expressed as

A19) $$\frac{dP}{dA_s} = \frac{1}{p_s}\left[1 + \frac{\alpha_m}{p_m}\left[e_{ms} - p_m\right] + \frac{\alpha_f}{p_f}\left[\frac{e_{fm}e_{ms}}{p_m} - p_f\right]\right]$$

When combing (A19) with the condition for the price effect of a shock to the demand for starter homes to equal the market segment's structurally adjusted size effect, $\frac{dP}{dA_s} = \frac{\alpha_s}{p_s}$, we find the market structure that eliminates the short run deviation in the PTI-ratio as

A20) $$\alpha_s = \left[1 + \frac{\alpha_m}{p_m}\left[e_{ms} - p_m\right] + \frac{\alpha_f}{p_f}\left[\frac{e_{fm}e_{ms}}{p_m} - p_f\right]\right],$$

which again equals

A21) $$\alpha_s = 1 - \alpha_m\left[1 - \frac{e_{ms}}{p_m}\right] - \alpha_f\left[1 - \frac{e_{ms}e_{fm}}{p_m p_f}\right].$$

3d. A heterogeneous housing market where shocks are homogenous

The partial derivative of (8) with respect to a demand shock equal across market segments is equal to:

A22) $$\frac{dP}{d\bar{A}} = \frac{\alpha_f}{p_f} + \left[\frac{\alpha_m}{p_m} + \frac{\alpha_f}{p_f}\left[\frac{e_{fm}}{p_m}\right]\right] + \left[\frac{\alpha_s}{p_s} + \frac{\alpha_m}{p_m}\left[\frac{e_{ms}}{p_s}\right] + \frac{\alpha_f}{p_f}\left[\frac{e_{fm}}{p_m}\frac{e_{ms}}{p_s}\right]\right]$$

Inserting for $\frac{dP}{d\bar{A}} = 1$, and using the fact that $\alpha_s + \alpha_m + \alpha_f = 1$, gives us the market structure necessary for no short run deviations in the PTI-ratio as

A23) $$\alpha_s = p_s\left[1 - \frac{\alpha_m}{p_m}\left(1 + e_{ms}\right) - \frac{\alpha_f}{p_f}\left(1 + \frac{e_{fm}}{p_m} + \frac{e_{fm}e_{ms}}{p_m p_s}\right)\right]$$