




Probabilistic Optimization Techniques in Smart Power System

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Abstract: Uncertainties are the most significant challenges in the smart power system, necessitating the use of precise techniques to deal with them properly. Such problems could be effectively solved using a probabilistic optimization strategy. It is further divided into stochastic, robust, distributionally robust, and chance-constrained optimizations. The topics of probabilistic optimization in smart power systems are covered in this review paper. In order to account for uncertainty in optimization processes, stochastic optimization is essential. Robust optimization is the most advanced approach to optimize a system under uncertainty, in which a deterministic, set-based uncertainty model is used instead of a stochastic one. The computational complexity of stochastic programming and the conservativeness of robust optimization are both reduced by distributionally robust optimization. Chance constrained algorithms help in solving the constraints optimization problems, where finite probability get violated. This review paper discusses microgrid and home energy management, demand-side management, unit commitment, microgrid integration, and economic dispatch as examples of applications of these techniques in smart power systems. Probabilistic mathematical models of different scenarios, for which deterministic approaches have been used in the literature, are also presented. Future research directions in a variety of smart power system domains are also presented.

Keywords: probabilistic optimization; stochastic optimization; robust optimization; distributional robust optimization; chance constrained optimization; energy management; smart grid



Citation: Riaz, M.; Ahmad, S.; Hussain, I.; Naeem, M.; Mihet-Popa, L. Probabilistic Optimization Techniques in Smart Power System. *Energies* **2022**, *15*, 825. <https://doi.org/10.3390/en15030825>

Academic Editor: Abdelali El Aroudi

Received: 20 November 2021

Accepted: 19 January 2022

Published: 24 January 2022

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1. Introduction

Energy demand is expanding in lockstep with global population expansion, resulting in a supply-demand mismatch. Increased generation capacity or load reduction can help close the demand-supply imbalance. Increased generation capacity is possible through the use of fossil fuels, which are costly and pollutant [1]. Enhancing generation capacity through the integration of renewable energy resources into the smart power system is beneficial. Load curtailment creates user displeasure, which can be mitigated by implementing appropriate demand-side policies. The integration of renewable energy supplies and variable load introduces a number of risks into the smart power system that must be handled. This article discusses uncertainty in a variety of sectors related to smart power systems. This is explained in greater detail below.

1.1. Smart Power System

Conventional grid electrical power is sent unidirectionally from a central power station to remote users. In 2000, a smart power system concept was established with the primary goal of integrating two-way communication into the infrastructure of a standard grid system. From the generating station to the consumers, a smart power system integrates

communication and information technology [2,3]. Consumers receive safe, robust, and high-quality power from a smart power system [4–6]. To transform a traditional grid into a smart power system, a scalable yet robust communication architecture is necessary [7]. In an electrical power system, the grid is composed of multiple energy generating, transmission, distribution, and control elements. The smart power system intelligently organizes and connects the traditional grid's elements previously stated [8–10].

In a smart power system, generating stations serve as the primary unit. Energy from renewable sources is required for new power plants since fossil fuels are becoming depleted and have other negative effects on the environment. Solar and wind energy have unpredictable output power since they are weather-dependent; as a result, the operation of smart power systems is affected as mentioned in [11–13]. Electrical power supply relies heavily on transmission systems because generating units are located distant from the end-users of the electricity. Climate change has a direct impact on the transmission system, resulting in issues such as wind and temperature stress. These uncertainties have a significant impact on the transmission system's performance and life expectancy [14].

Intelligent distribution systems are a critical component of a smart power system. It is composed of a variety of sensors and smart sensing mechanisms, as well as a sensor network that includes smart metres, distribution transformer management, and monitoring. Due to the usage of these sensors, a smart grid can monitor the health of the grid in real time and utilize the data to operate the grid in a reliable, secure, and stable manner, while also reducing costs and increasing energy efficiency. Sensors and actuators play a critical role in the energy management of smart power systems in this context [15,16]. Employing sensors for real-time monitoring helps to deal with the power management of distributed energy resources, such as distributed generators (DGs) [17] and electric vehicles [18], as well as to improve the smart grid's reliability as it makes the integration of renewable energy resources much more convenient and improves both the efficiency and reliability of smart power system [19,20]. Due to the integration of distributed generation, the smart distribution system is unpredictable. It may have uncertainty due to the fault resistance, the magnitude and model of the load, the faulted node, and the type of fault [21].

Energy management is a critical component of the advancement of smart power systems. It is applicable to a variety of smart power system domains, including microgrids, smart homes, and demand side management. Due to the unpredictable behaviour of renewable energy resources and loads, energy management challenges face an uncertainty challenge [22]. The complexity of unit commitment and economic dispatch problems increases with the inclusion of uncertainties [23,24]. It is necessary to investigate probabilistic optimization in order to formulate the influence of various types of uncertainty in intelligent power systems.

1.2. Related Work and Contributions

Numerous survey papers from the literature have been meticulously analysed and summarized for comparison in this review paper, as indicated in Table 1. In [25], the authors discussed stochastic optimization and offered architectures and solution techniques for single and multistage stochastic optimization. The architecture of stochastic and chance restricted optimization is demonstrated in [26]. Additionally, applications of stochastic and chance constrained optimization in a variety of industries have been summarized, including banking, transportation, telecommunications, and manufacturing. The [27] discusses stochastic and chance constrained optimization for intelligent power systems. Additionally, the authors addressed many variants of the above-mentioned optimization techniques' objective function and architecture. The authors of [28] described the architecture of robust optimization and highlighted its applications in a variety of fields, including structural design, circuit design, and wireless channel design. In [29] discusses the architecture and solution strategies for robust optimization. The authors discussed distributionally robust optimization, its architecture, and categorization of ambiguity sets in [30]. In [31], they discussed the design of single and two stage chance constrained optimization. This review

paper differs from the survey papers that are already available in the literature in a number of ways, which are detailed below.

- It gives a complete review of stochastic, robust, distributionally robust, and chance restricted optimization in the domain of smart power systems in a single survey study.
- An overview of numerous probabilistic optimization strategies, including their taxonomy, application examples, and solution algorithms is included in this survey study.
- Probabilistic mathematical models for various scenarios that can be used as a reference models in the field of smart power system have been developed.

Table 1. Summary of Related Survey Papers.

Ref.	SO	RO	DRO	CC	AR	TN	OF	SA	Smart Power System
[25]	✓				✓			✓	
[26]	✓			✓	✓				
[27]	✓			✓	✓		✓		✓
[28]		✓			✓				
[29]		✓			✓			✓	
[30]			✓		✓				
[31]				✓	✓				✓
Our Review Paper	✓	✓	✓	✓	✓	✓	✓	✓	✓

1.3. Organization of the Paper

This paper is organized as follows: Section 1 introduces the smart power system, its elements and related research contribution, while Section 2 covers the architecture and taxonomy of probabilistic optimization. Applications, objectives and solution algorithms of probabilistic optimization in various domains of smart power system are discussed in Section 3. Section 4 furnishes probabilistic mathematical models of various scenarios in smart power system. Then, future research directions and new challenges are discussed in Section 5, whereas Section 6 provides a brief summary of the whole article with concluding remarks.

2. Probabilistic Optimization

Stochastic programming is used to solve optimization problems in which the majority of the parameters are probabilistic [32]. Probabilistic optimization can make efficient use of information, both in terms of selecting evaluation points and the message they convey. It can handle many sorts of noise and adapts to various aspects of optimization issues. Unlike deterministic optimization, probabilistic optimization techniques discover the best solution for data with randomness [33]. As indicated in Figure 1, there are multiple probabilistic optimization categories: stochastic optimization, robust optimization, distributionally robust optimization, and chance-constrained optimization.

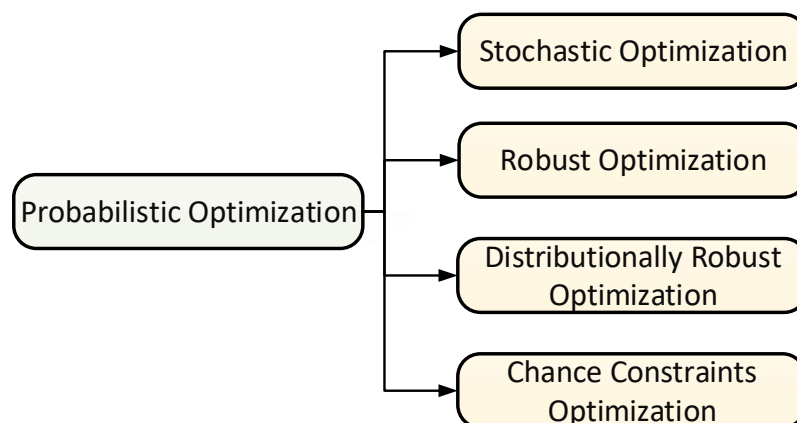


Figure 1. Classification of probabilistic optimization.

2.1. Stochastic Optimization

Stochastic optimization is critical for addressing uncertainty in optimization problems. Due to computing problems, uncertainty is typically disregarded in classical optimization, but breakthroughs in computational techniques now allow for the efficient handling of uncertainties [34]. Stochastic optimization is concerned with strategies for minimising or optimising an uncertain objective function. In contrast to deterministic optimization issues, stochastic optimization problems do not have a single solution. To solve the issue tractably, structural assumptions such as a constraint on the size of the choice variables, the result space, or convexity are required [35]. Traditionally, stochastic optimization modeled uncertainties as random variables with well-defined distributions [36].

2.1.1. Architecture of Stochastic Optimization

The objective function is typically optimized over the expected value of the uncertain parameters for the formulation of stochastic programming, as shown in Equation (1). Where x is the decision variable that belongs to set X , E_p is the expected value of the random variable ξ . Stochastic optimization is graphically represented in part *a* of Figure 2, where P is the probability distribution of random variable ξ . An exact distribution is required for the uncertainties, which cannot be estimated with the empirical data accurately [37]. Either all scenarios or scenarios with probability guarantees are feasible for the modeled solution. In stochastic optimization, sample-based techniques are commonly utilized due to the difficulty of obtaining the correct distribution of random variables. A greater sample size is utilized to get higher probability guarantees, increasing computing complexity [34].

$$\begin{aligned} \inf_x E_p\{f(x, \xi)\} \\ \text{s.t. } x \in X \end{aligned} \quad (1)$$

The probability distribution determines the level of uncertainty in stochastic optimization. In basic scenarios, uncertainty is well known, but in practise, it is only partially unknown. The accuracy of stochastic optimization is influenced by the model specifics and availability of possible scenarios. If a stochastic framework is used for all scenarios, the problem becomes more difficult. A trade-off between number of scenarios and, computing time, and complexity is required [34].

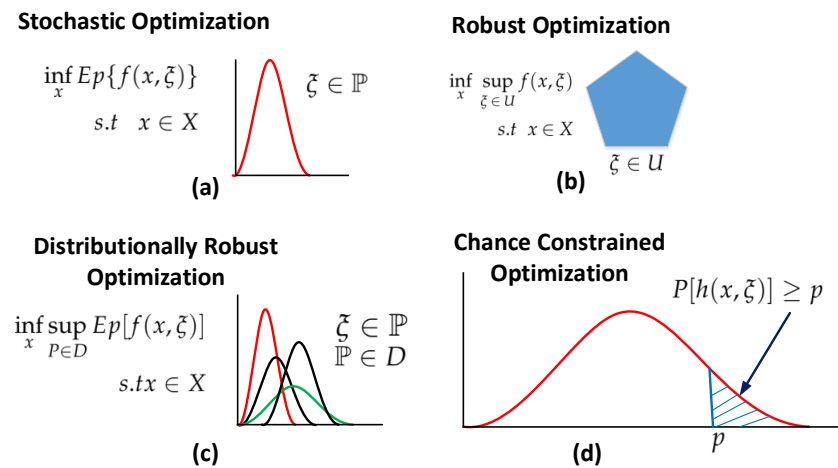


Figure 2. Graphical comparison of: (a) stochastic optimization; (b) robust optimization; (c) distributionally robust optimization; (d) chance constrained optimization.

2.1.2. Taxonomy of Stochastic Optimization

Stochastic optimization can be categorized into single stage problems and recourse problems. The recourse problems can be further classified into two stage and multistage problems as shown Figure 3 [25]. In single stage problems, a single but optimal decision is obtained where in recourse optimization problems, it is essential to know the probability distribution of the random variable in the first step, where the second step (correction of that decision) is being performed. In the two stage stochastic optimization the decision maker must make judgments in two stages (at two distinct times) for a given phenomenon with uncertainty. The first stage choice is critical since it must be made based on some random factors gleaned from previous experience or a survey.

Two-stage stochastic optimization problems may have fixed recourse or complete recourse. In case of fixed recourse, the first stage is prediction stage where in second-stage, fixed decision is done based on the results of the experiment [38]. Two-stage stochastic optimization problems will be considered as a complete recourse if, for every scenario, there always exists a viable second solution [33]. Multistage stochastic programming is an extension of two stage stochastic programming to the sequential realization of uncertainty. Majority of the real time problems lies in the domain of multistage stochastic optimization which entail a series of decisions in response to changing outcomes over time [35].

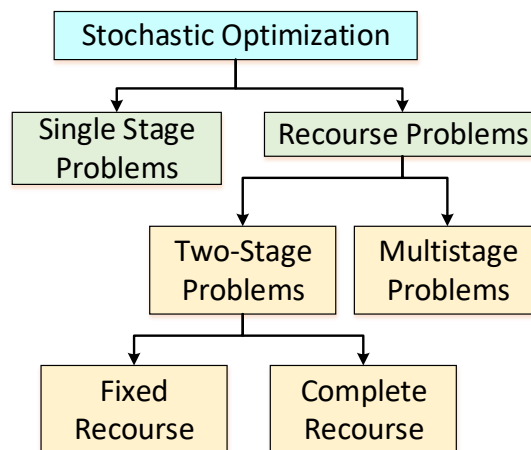


Figure 3. Taxonomy of stochastic optimization.

2.2. Robust Optimization

Robust optimization is a relatively new technique for optimising in the presence of uncertainty. Rather than using a stochastic model, it uses a deterministic, set-based uncertainty model. The robust optimization solution is valid for any specification of the uncertainty in a given set. The reason for robust optimization is that it accounts for both set-based uncertainty and computational tractability [28]. Robust optimization and the respective computational tools deal with optimization problems in which the information are indeterminate and belong to some set of uncertainty [39]. Robust optimization ensures that the worst-case scenario is realized, ensuring that the solution is both practical and optimal for a given set of uncertainties. Robust optimization is not chosen in some applications due to its conservative nature, however it is used in the power industry to preserve reliability. Robust optimization necessitates a considerable amount of knowledge about the uncertainty, such as its size and range [34].

2.2.1. Architecture of Robust Optimization

Robust optimization is a realization of worst-case parameters that belong to the uncertainty set. Worst case realization of robust optimization sometimes becomes unrealistic in practice [37]. The architecture of robust optimization is available in Equation (2), where x is the decision variable that belongs to set X , ξ is a random variable belonging to the uncertainty set U . Robust optimization can be graphically represented as shown in part *b* of Figure 2.

$$\begin{aligned} \inf_x \sup_{\xi \in U} f(x, \xi) \\ \text{s.t. } x \in X \end{aligned} \quad (2)$$

2.2.2. Taxonomy of Robust Optimization

Robust optimization can be categorized as shown in Figure 4, and is discussed as follows.

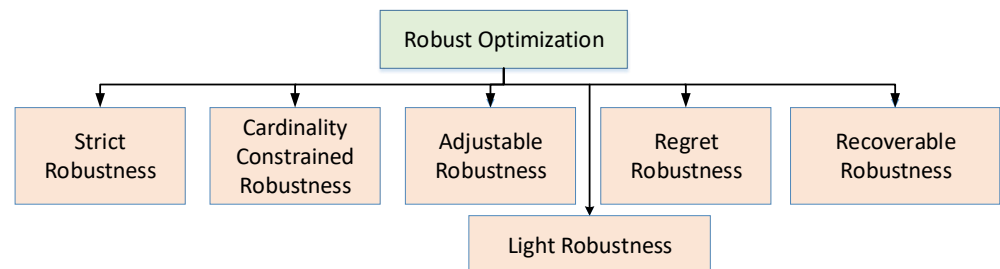


Figure 4. Taxonomy of robust optimization.

- **Strict robustness:** This optimization type is sometimes known as classic robust optimization, min–max optimization, absolute deviation, one-stage robustness, or simply robust optimization. It is treated, as the fundamental starting point in the area of robustness. A solution x is called strictly robust if it is feasible for all possible scenarios of uncertainty set U [40].
- **Cardinality constrained Robustness:** In cardinality constrained robustness, reduction in uncertainty’s space can relax strictness in robust optimization. Analyzing the worst-case scenario in robust optimization, it is improbable that all the uncertainty set parameters will change simultaneously. Hence, it restricts uncertainty space by varying some parameters while considering fixed values for the remaining [41].
- **Adjustable robustness:** In adjustable robustness, the uncertainty space of strict robustness gets relaxed by dividing uncertainty space into groups of variables such as here and now and wait-and-see. Variables from the here and now group must be evaluated

before the scenario $\zeta \in U$ is determined where variables from the wait-and-see group can be determined once the scenario ζ is known [42].

- Light robustness: In light robustness, relaxing the constraints in terms of quality can reduce the strictness of the robust optimization, rather than reducing the space of uncertainty. Light robustness develops a trade-off between quality and robustness of the solution [43].
- Regret robustness: In regret robustness, the objective function relaxes the problem. Rather than to minimize the worst case performance of the solution, regret robustness reduces the difference of objective function having the best solution and the objective function that would have been possible in a scenario [44].
- Recoverable robustness: Concept of recovery algorithm gets exploited in recoverable robustness and family of recovery algorithms which is represented by B . It provides the solution in two stages, such as adjustable robustness. A solution x is called recovery robust with respect to recovery algorithm A if for any probable situation $\zeta \in U$ an algorithm $A \in B$ exist such that when A is applied to the solution x and the scenario ζ makes a solution $A(x; \zeta) \in F(\zeta)$ [45].

2.3. Distributionally Robust Optimization

Distributionally robust optimization, also known as min–max stochastic programming, reduces the computational complexity of stochastic programming and conservative nature of robust optimization. It turns up optimal decisions for the worst-case probability distributions within a family of possible distributions, defined by specific characteristics such as their support vector and moments information [36]. As compare to stochastic programming, it is less dependent on the data having an exact probability distribution. Due to the incorporation of probability distribution and concept of ambiguity sets, the result becomes less conservative as compared to simple robust optimization [46].

2.3.1. Architecture of Distributionally Robust Optimization

A distributionally robust optimization or min–max stochastic programming model act as a bridge between robust and stochastic optimization. It usually takes the form, as shown in Equation (3). where x is the decision variable that belongs to set X , P is the probability distributions that belongs to an ambiguity set D , ζ is the random variable, and Ep is expected value of the random variable [37]. Distributionally robust optimization can be graphically represented as shown in part *c* of Figure 2. The random variable ζ belongs to probability distribution P where P itself belongs to ambiguity set D .

$$\inf_x \sup_{P \in D} Ep[f(x, \zeta)] \quad (3)$$

$$s.t. x \in X$$

2.3.2. Taxonomy of Distributionally Robust Optimization

Various categories of distributionally robust optimization (DRO) are shown in Figure 5 and are discussed as follows. DRO is a strong modeling paradigm for optimization under uncertainty that arises from the realization that the probability distribution of uncertain parameters of the the problem is uncertain in-itself. As a result the concept of ambiguity set arises which is defined as a set in which the modeler considers that the real distribution of the uncertain parameters of problem has uncertainty. Naturally, the ambiguity set's creation is critical to DRO's actual effectiveness. DRO can be classified on the basis of characteristics and specifications of ambiguity set which are described as follows [47].

1. Moment-based approach: The ambiguity set in moment based approach is the set of all probability distributions whose moments satisfy certain constraints [48,49].
2. Dissimilarity-based approach: The ambiguity set in this case is the set of all probability distributions whose dissimilarity to a nominal distribution is lower than or equal to a

given value. In this category, the choice of the dissimilarity function leads to couple of different variants which are as follows [47].

- (a) Optimal-transport-based (OTP) approach: The authors in [50,51] applied Wasserstein distance as a dissimilarity function which shows some nice statistical convergence properties.
- (b) ϕ -Divergences based approach: This group consist of all those techniques which uses ϕ -divergences such as Kullback–Leibler divergence, as was described in [52,53]. Approaches used in [54,55] are based on likelihood which also belongs to this category.

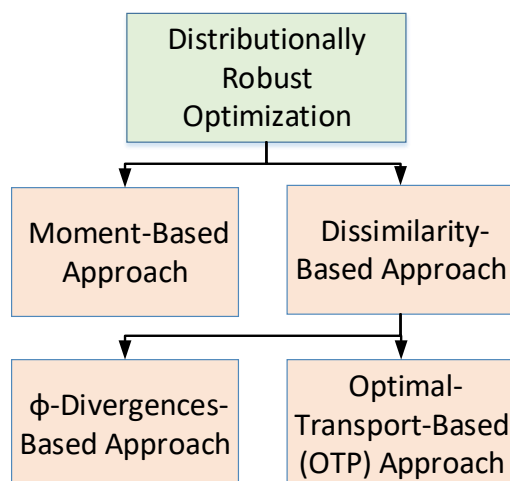


Figure 5. Taxonomy of distributionally robust optimization.

2.4. Chance Constrained Optimization

Chance constrained optimization solves the problems having constraints, in which finite probability get violated. As compared to conventional optimization problems, chance constrained optimization problems face a challenge when inequality function is not available explicitly. Hence, no suitable algorithmic or theoretical properties are evident, such as differentiation, continuity and concavity. A general solution method for chance constrained programming does not exist, but it depends on the interaction of decision and random variables in the constraint model [56].

2.4.1. Architecture of Chance Constrained Optimization

In general chance constrained optimization can be expressed as in inequality, as shown in Equation (4), where it can be graphically represented as shown in part *d* of Figure 2:

$$P[h(x, \xi)] \geq p \quad (4)$$

In Equation (4), ξ and x are random and decision vector, respectively, P is probability measure, $h(x, \xi) \geq 0$ represents a finite system of inequalities. $p \in [0, 1]$ is known as probability level and is chosen by decision maker for safety requirements [56].

2.4.2. Taxonomy of Chance Constrained Optimization

Chance constrained optimization problems can be categorized based on constraints involved as shown in Figure 6. It may have individual, joint, or mixed chance constrained. In individual chance-constrained optimization problems, each element of the stochastic inequality system is transformed into several chance constrained in a unique way where in joint chance-constrained optimization problems, the probability is considered over the stochastic inequality system as a whole. Chance constrained optimization in Equation (4) can be expressed as an individual and joint chance-constrained, as shown

in Equations (5) and (6), respectively, [56]. Mixed chance constrained optimization problems may comprise numerous multivariate chance constrained [57]. Individual chance constrained are simple but unreliable compared to joint chance constrained; hence joint chance constrained are used to guarantee the decision at a given probability level [56].

$$P[h_j(x, \xi)] \geq 0 \geq p_j (j = 1, 2, 3, \dots, m) \quad (5)$$

$$P[h_j(x, \xi)] \geq 0 \quad (j = 1, 2, 3, \dots, m) \geq p \quad (6)$$

Based on the constraints involved in chance constrained optimization problems, it may be linear random vector, separated random vector, coupled random vector, or decision vector. Most important model of chance constrained system is linear random vector. Linear random vector is shown in Equation (4) where the constraint h may adopt different form such as expressed in Equations (7) and (8).

$$h(x, \xi) = g(x) - A \cdot \xi \quad (7)$$

$$h(x, \xi) = A(\xi)g(x) - b \quad (8)$$

where $A(\xi)$ and A are stochastic and deterministic matrices, respectively, b is a constant vector of suitable size, g is the function of decision vector x . The model shown in Equations (7) and (8) represents separated and coupled random vector, respectively. In isolated random vector, random vector and decision vector appear separated while combined in the coupled vector model. The random vector may be continuous, discrete, independent or correlated [56].

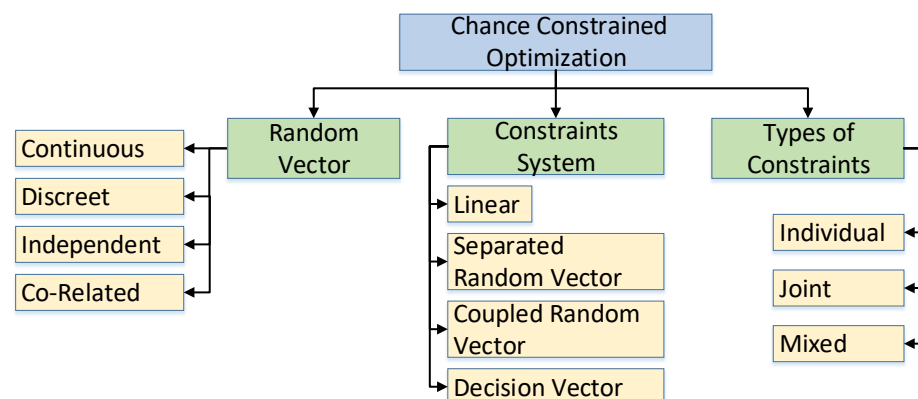


Figure 6. Taxonomy of chance constrained optimization.

3. Applications, Objectives and Solution Algorithms of Probabilistic Optimization

3.1. Applications, Objectives and Solution Algorithms of Stochastic Optimization

Applications of stochastic optimization shown in Table 2. Microgrid energy management problems might be seen as having uncertainty in plug-in electric vehicles and distributed renewable energy supplies [58]. The authors in [59] used stochastic dynamic programming to analyse smart home energy management with uncertainty in plug-in electric vehicles. The optimization problem is approached as non-linear programming, and the distribution of electric power among various smart home components is optimized.

The energy management problem of a smart thermal grid with aquifer thermal energy storage is solved using a stochastic model predictive control framework. Mixed-integer quadratic programming is used to solve the problem. The developed model is used to capture the aquifer's injection and extraction imbalances, as well as the undesired mutual interaction of aquifer thermal energy storage and smart thermal grid [60]. The problem of probabilistic optimal power dispatch for microgrid is defined as non-linear programming. The operating cost is minimized through particle swarm optimization, and the optimization

problem is handled appropriately [61].

In day-ahead transmission network planning, a probabilistic model is utilized to schedule demand response. For the best demand response scheduling, network security and consumer economic factors are applied. Thermal units and renewable energy resources have been modeled, and the problem has been formulated using mixed-integer linear programming [62]. Residential appliances use real-time demand response management with stochastic and robust optimization. The mathematical model is developed using mixed-integer linear programming, and the electricity bill is reduced as compared to flat rate [63].

The probabilistic model has better performance to solve the smart power system economic dispatch problem as compare to the negative load reduction model for various cases [23]. Economic operation of future distribution grid is discussed, and the stochastic model is developed to find the optimal operation of small-scale energy resources and load [64]. Non-linear programming is used to develop mathematical model of the system as described in [23,64].

A stochastic model is utilized to reconfigure the distribution grid and model distributed photovoltaic generation in this study. The distribution grid is operated at its most cost-effective level, and different constraints such as power balance and power flow limits are met. The grid's reliability and stability have also been improved, which is an important component of incorporating renewable energy sources. Mixed-integer second-order cone programming is used to solve the optimization problem [65].

The stochastic optimization method is used to solve a unit commitment problem with a demand response that is uncertain. It is demonstrated that by taking the uncertainty of demand response into account in a probabilistic manner, generating capacity may be enhanced [66]. The model was created to deal with uncertainty in the unit commitment problem while minimizing the system's operating costs. To acquire an efficient solution for the system, parallelization and decomposition strategies are applied [67]. The integration of storage devices and a high penetration of renewable energy resources is demonstrated in a unit commitment and economic dispatch model. It is concluded that the consideration of storage devices to reduce operational cost is quite effective. Linear programming is used to formulate optimization problem in [66,67], where in [24] mixed-integer linear programming is applied.

For day-ahead optimal power flow, stochastic optimization is used, which can help to improve economic benefits. The uncertainties of wind power and load are incorporated into DC optimal power flow. The problem is expressed as a mixed integer linear programming problem that is solved using the two-point estimation approach. To determine the ideal number of switching each hour, a framework based on probability decisions is designed, taking into account risk cost and economic rewards [68].

Table 2. Applications of stochastic optimization and its problem type.

References	Applications	LP	NLP	MILP	MISOCP	MIQP
[59]	HEM		×			
[61]	MEM		×			
[68]	OPF			×		
[62,63]	DRM			×		
[23,64]	ED		×			
[24,66]	UC	×		×		
[60]	STG					×
[65]	RDG				×	

3.2. Applications, Objectives and Solution Algorithms of Robust Optimization

In a smart power system, robust optimization has a variety of applications with dynamic objectives. Table 3 shows the applications, and Table 4 summarizes the objectives.

3.2.1. Smart Grid Energy Management

One of the most common applications of robust optimization is smart grid energy management. Robust optimization can be used to model uncertainty in several parameters. To maximize social welfare, the problem is described as mixed integer linear programming and solved using a consensus algorithm and an optimal control technique [69].

3.2.2. Microgrid Energy Management

The energy management system for single and three-phase balance microgrids is designed using robust convex optimization. The problem is formulated as a mixed integer second-order cone programming [70]. Microgrid energy management takes into account the characteristics and constraints of the system. The system is mathematically modeled using mixed-integer nonlinear programming, which is subsequently linearized using the Lyapunov optimization approach [71]. For microgrid energy management, two-stage adaptive robust optimization is employed while taking into account the uncertainties of renewable energy resources. In both isolated and grid connected modes, the problem is formulated as mixed integer linear programming, and the total operating cost of the system is minimized. The column and constraint creation algorithm efficiently solves the problem [72]. Energy and frequency management of microgrid accomplishes a reliable and robust solution where total cost of the system is minimized by solving mixed integer linear programming using information gap decision theory [73,74].

Microgrid planning uses two-stage robust optimization to reduce operating and maintenance costs, investment costs, emissions, and fuel costs. The composition of a microgrid takes into account both renewable energy sources and dispatchable distributed generation. The key sources of uncertainty are intermittent renewable energy resources and time-varying load, which can be effectively managed by using robust optimization. The column and constraint creation approach aids in the solution of the mixed integer linear programming problem [75]. Scenario-based robust optimization is applied to minimize the microgrid's social benefit cost by accounting for uncertainty in load and renewable energy resources. Taguchi's orthogonal array generates scenarios, which are then verified using Monte Carlo simulations [76]. Distributed generation, distributed storage, and distributed economic dispatch are used to manage energy in the microgrid. The Lagrangian relaxation and dual decomposition method is used to reduce the net cost of a microgrid [77].

3.2.3. Unit Commitment

A security constraint unit commitment for the power grid considering the uncertainties in supply and demand is performed. Total operation cost is minimized, and the solution to the problem is achieved by applying the bender's decomposition and column generation methods [78]. The overall cost of the system gets minimized by applying various algorithms in different domains [79,80]. Unit commitment problem is solved by Benders' decomposition algorithm [81,82], column and constraints generation algorithm [83,84] and Lagrangian decomposition method [85]. Integrated electricity and heating system is scheduled by column and constraints generation algorithm [86]. Multistage robust optimization is applied for unit commitment, considering the uncertainties of wind power and demand response. The sole objective is to maximize social welfare and to satisfy various constraints. It is being solved by using bender's decomposition algorithm to achieve unit commitment in an optimal robust way [87].

3.2.4. Demand Side Management

The demand side is scheduled using robust optimization, which takes into account the uncertainty in manually operated appliances. The problem is formulated as quadratic programming [88], where nonlinear programming is used in [89] to minimize the cost of electricity. Commercial building appliances are scheduled in an ideal method to account for the impact of uncertainties. To minimize the cost of power, the optimization problem is framed as a mixed integer linear programming problem.

3.2.5. Smart Home

The robust index method is applied to handle the uncertainties of household load scheduling and minimize the customer discomfort. The problem is mathematically formulated as a mixed integer linear programming which has been solved by using branch and bound algorithm [90]. The proposed model schedules renewable energy resources at the production part and controls the smart home consumption part. An optimal solution achieved, along with the reduction in computational time and electricity cost. Meta-heuristic algorithm is applied to solve the mixed integer linear programming problem [91].

3.2.6. Plugin Electric Vehicles

Bidirectional dispatch coordination of plugin electric vehicles in a power grid restrains the generation cost. The problem is formulated as a mixed integer linear programming and solved by using heuristic approach [92].

Table 3. Applications of robust optimization and its problem Type.

Ref.	Applications	LP	NLP	MIP	MILP	MINLP	MIBLP	MISOCP	MIQP	QP
[69]	SGEM				×					
[70–73,75]	MEM				×	×		×		
[90,91]	HEM				×					
[88,89,93]	DSM		×		×					×
[92,94]	PEV				×				×	
[67,78,83–85,87,95–104]	UC	×	×	×	×		×		×	
[105]	SGTD				×					

3.3. Applications, Objectives and Solution Algorithms of Distributionally Robust Optimization

When considering energy storage, distributed generators, and wind turbines, distributionally robust chance constrained programming is used for energy management of an islanded microgrid. However, using an analytical method, the overall generation cost is minimized [106]. The generation frequency is managed appropriately via quadratic programming, and the generation cost is also curtailed [107].

The unit commitment problem with uncertainty in wind output power is solved via distributionally robust optimization. The MILP optimization problem is solved using an analytical method, and conservatism is reduced by using distribution information. [46,108]. Distance-based distributionally robust optimization is modeled for a unit commitment by using Kullback Leibler divergence. This model handles uncertainties of wind power in the form of an ambiguity set. The problem is arranged as mixed integer non-linear programming and is solved by using bender decomposition and the iterative method. Computational complexities are handled by decomposition method while the iterative algorithm guarantees the global conservatism [109]. The unit commitment problem is solved by Benders' decomposition algorithm [110].

The authors in [112] applied distributionally robust optimization to solve energy and reserve dispatch problem. It is shown that distributionally robust optimization is a suitable technique for reserve dispatch to fill the gap between stochastic and adjustable robust optimization. Strategic aggregation is offering regulation capacity on behalf of a group of distributed energy resources. Two stage stochastic optimization and distributionally robust chance constrained optimization are utilized for handling the uncertainties in day-ahead and hour-ahead schemes, respectively, [113]. The authors in [114] applied distributionally robust optimization to solve the power flow problem.

3.4. Applications, Objectives and Solution Algorithms of Chance Constrained Optimization

Chance constrained optimization can be applied to a smart power system by considering applications with diverse objectives. Applications and objectives of chance constrained optimization in smart power system are shown in Tables 5 and 6, respectively.

3.4.1. Microgrid Energy Management

Chance constrained optimization for microgrid energy management is used, where uncertainties are considered in various parameters. Electricity cost of microgrid is minimized by using linear programming while satisfying the energy balance constraint [115]. In [116], chance constrained optimization is applied to handle the uncertainties in power exchange between microgrid and macro-grid where overall cost of the system is minimized by using mixed integer linear programming. Chance constrained stochastic cone programming is applied to plan microgrid network and overall system's cost is minimized. To obtain the solution for the problem, it uses second-order cone programming (SOCP), bi-linear Benders decomposition method, Jensen's inequalities, and Pareto-optimal cuts [117]. Chance constrained optimization is used for the optimal operation of microgrid having uncertainties where the problem is formulated as a mixed-integer non-linear programming [118].

3.4.2. Distributed Energy Management

In the distribution system, chance constrained optimization helps in the operation and planning of the energy storage system. Overall cost of the system is minimized by using mixed integer linear programming [119,120]. Chance constrained optimization is used to handle the uncertainties that are due to photo-voltaic and batteries. Line losses in the distribution system are reduced by formulating the problem as second order cone programming and solved by analytic method [121,122]. Overall cost of the system is minimized in distributed energy management problem by using mixed integer linear programming [123]. In [124], the authors presented feasibility and profit based planning for the integration of distributed generation. The problem is mathematically formulated as a mixed integer bi-linear programming.

3.4.3. Demand Side Management

Uncertainties due to the consumption pattern and variation in consumers response to the price signal are modeled by chance constrained optimization. The problem is mathematically formulated as non-linear programming to minimize the electricity price and being solved by interior point method [125]. In [126], the authors considered uncertainties due to the interruptable load and consumer response. Penalty to the consumers and variations due to the interrupt-able load are minimized using non linear programming.

3.4.4. Smart Distribution Network

Joint chance constrained optimization handles the high penetration of distributed generator in a distribution network. The support vectors classifier (SVC) identifies zero probability constraints while sampling is done by Monte Carlo Simulations. Overall system cost is reduced by using non-linear programming to formulate the problem [127]. In [128],

the authors minimized planning cost where the problem is being formulated as mixed integer non linear programming.

3.4.5. Home Energy Management

Chance constrained optimization for home energy management to optimize the operation of appliances is used. The model to formulate the uncertainties due to electricity prices and fluctuating loads is used. The problem is mathematically modeled as mixed-integer linear programming and being solved by using particle swarm optimization and two-point estimation method [129].

3.4.6. Unit Commitment

The chance constrained two stage stochastic program minimizes the overall generation cost, whereas sample average approximation helps in solving the mixed integer linear programming problem [130]. Spinning reserve cost gets minimized in an uncertain controllable load by using chance constrained optimization. The problems are mathematically formulated as linear programming which are being solved by applying the analytic method and scenario base analysis in [131] and in [132], respectively. Overall system's cost gets minimized by applying the ranking algorithm, and the iterative method in unit commitment problem using mixed integer linear programming [123]. The authors in [133] applied analytic method to satisfy the constraints in unit commitment problem. Operating cost is minimized by formulating the unit commitment problems as mixed integer programming and mixed integer second order cone programming in [134,135], respectively. Overall cost of the system is minimized in [136] using mixed integer quadratic programming and non-linear programming in [137].

3.4.7. Economic Dispatch

Economic dispatch problem is formulated as a linear programming problem in [138]. Active power losses are minimized in thermostatically controllable load, where Spatio temporal and dual decomposition algorithm solve the problem [139]. The pay-off gets maximized, and the problem is solved by applying linear regression and iterative method [140]. Dispatch coordination for plug-in electric vehicles are modeled as a mixed integer quadratic programming [141].

Table 5. Application of chance constrained optimization and its problem type.

Ref.	Applications	LP	NLP	MIP	MILP	MINLP	MIBLP	MISOCP	SOCP	MIQP
[115–118,142,143]	MEM	×	×		×	×		×		
[129]	HEM				×					
[119–121,123,124]	DEM				×		×		×	
[127,128]	SDN		×			×				
[125,126]	DSM		×							
[141]	PEV									×
[138]	ED	×								
[130,131,133–137,144]	UC	×	×	×	×			×		×
[145]	GEM		×							
[146]	OPF		×							
[147]	OPGF		×							

4. Mathematical Models for Various Scenarios

This review paper presents probabilistic mathematical models of diverse scenarios for better understanding of scientists and researchers. Researchers have, however, presented deterministic mathematical models in the literature.

4.1. Scenario 1: Energy Management

In [150], the authors presented an energy management model for a residential compound having N number of consumers, as shown in Figure 7. Each consumer has a set of appliances A , which includes seasonal, shift-able and non shift-able appliances. The system has some given parameters, but some parameters are to be determined.

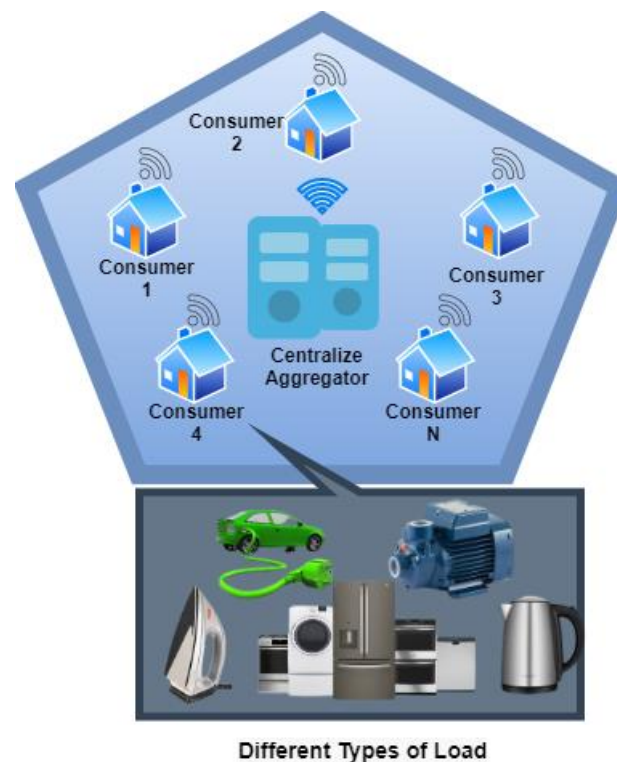


Figure 7. Residential compound.

Given Parameters:

- Total number of consumers N in residential compound
- Set of appliances A for each consumers
- Each appliances has a time dependent power profile $P_a^{t,n}$
- Each appliances operating time t_a^n
- Scheduled starting time t_s^n
- Human interaction factor for a certain time $H_a^{t,n}$
- Price tariff C^t
- Load shedding factor L^t

Parameters to be determined for each time slot

- To switched on a set of appliances
- Each consumer electricity consumption cost

A deterministic model of the system is shown in Equation (14), where the appliances are scheduled in such a way so that it reduces the consumer's electricity cost. To turn on the given appliance we have introduced binary decision variable as shown in Equation (9).

$$X_a^{t,n} = \begin{cases} 1 & \text{if the } a\text{th appliances of the } n\text{th} \\ & \text{consumer is switch on at the } t\text{th time slot.} \\ 0 & \text{otherwise} \end{cases} \quad (9)$$

The a th appliance of the n th consumer requires t_a^n minutes to operate. To accomplish the task, the scheduling time begins at $t = t_s^n$ and ends at $t = t_a^n + t_s^n$. The decision variable $X_a^{t,n}$ would be 1 for the complete number of t_a time slots to ensure continuous operation of the appliance. Hence, the contiguous constraint is represented in mathematical form as shown in Equation (10).

$$\begin{aligned} \sum_{t=t_s^n}^{t_s^n+t_a^n} X_a^{t,n} &= t_a^n, \quad \forall a, n \\ t_a^n + t_s^n &\leq T \quad \forall a, n \end{aligned} \quad (10)$$

Certain appliances require human involvement to operate, hence human interaction factor (HIF) is considered. The human interaction factor is represented by the symbol $H_a^{t,n}$ and is mathematically modeled as shown below.

$$H_a^{t,n} = \begin{cases} 1 & \text{if the } n\text{th consumer is available to operate the} \\ & a\text{th appliance at the } t\text{th time slot.} \\ 0 & \text{otherwise} \end{cases}$$

The operation of a washing machine, for example, necessitates the presence of consumers. As a result, the human interaction factor $H_a^{t,n}$ will be 1 for a given time interval to correctly operate the washing machine. By inculcating the impact of HIF factor, the Equation (10) becomes as shown in Equation (11).

$$\sum_{t=t_s^n}^{t_s^n+t_a^n} X_a^{t,n} H_a^{t,n} = t_a^n, \quad \forall a, n \quad (11)$$

Another concern related with residential energy management is the lack of energy owing to a variety of factors, particularly in rural locations, such as maintenance etc. We introduce the load shedding (LS) factor L^t to address the load shedding issue and the lack of electricity.

$$L^t = \begin{cases} 1 & \text{if the electricity is available for the whole time.} \\ 0 & \text{otherwise} \end{cases}$$

With the addition of load shedding factor, Equation (11) can be rewritten as shown in Equation (12).

$$\sum_{t=t_s^n}^{t_s^n+t_a^n} X_a^{t,n} H_a^{t,n} L^t = t_a^n, \quad \forall a, n \quad (12)$$

Another thing to consider, while building an effective residential energy management system, is the willingness and preferences of consumers (CP). There are times when it is ideal for a particular appliance to turn on, but the consumer is unwilling to operate or switch it on. As a result, we add another input variable, $\lambda_a^{t,n}$ to respond to consumer preferences.

$$\lambda_a^{t,n} = \begin{cases} 1 & \text{if the } n\text{th consumer wants to operate the } a\text{th} \\ & \text{appliance at the } t\text{th time slot.} \\ 0 & \text{otherwise} \end{cases}$$

With the consideration of consumer preference Equation (12) can be expressed as shown in Equation (13)

$$\sum_{t=t_s^n}^{t_s^n+t_a^n} X_a^{t,n} H_a^{t,n} L^t \lambda_a^{t,n} = t_a^n, \quad \forall a, n \tag{13}$$

$$\min_{X_a^{t,n} \in \{0,1\} \forall a,n} \sum_{n=1}^N \sum_{a \in A} \sum_{t=1}^T X_a^{t,n} C^t E_a^{t,n}$$

subject to

Contiguous Constraint

$$C_1 : \overbrace{\sum_{t=t_s^n}^{t_s^n+t_a^n} X_a^{t,n}} = t_a^n, \quad \forall a, n, t$$

LS and contiguous constraint

$$C_2 : \overbrace{\sum_{t=t_s^n}^{t_s^n+t_a^n} X_a^{t,n} H_a^{t,n} L^t} = t_a^n, \quad \forall a, n, t$$

$$C_3 : \overbrace{\sum_{t=t_s^n}^{t_s^n+t_a^n} X_a^{t,n} H_a^{t,n} L^t \lambda_a^{t,n}} = t_a^n, \quad \forall a, n, t$$

CP, LS, HIF and Contiguous Constraint

(14)

Considering that various parameters in Equation (14) have uncertainty such as total energy consumed $E_a^{t,n}$ and human interaction factor $H_a^{t,n}$. Uncertainty in $E_a^{t,n}$ is ζ_e^t while ζ_h^t is the uncertainty in $H_a^{t,n}$. To handle these uncertainties various probabilistic models are as follows.

4.1.1. Stochastic Optimization Model

Stochastic Optimization model is shown in Equation (15) where $\mathbb{E}_{\zeta_e^t}(E_a^{t,n}, \zeta_e^t)$ is the expected value of energy consumed at time T by each appliance of n^{th} consumer. Constraints C_1 in Equation (15) will remain same as shown in Equation (14). $[\mathbb{E}_{\zeta_h^t}(H_a^{t,n}, \zeta_e^t)]$ in C_2 and C_3 represent the expected value of human interaction factor.

$$\min_{X_a^{t,n} \in \{0,1\} \forall a,n} \sum_{n=1}^N \sum_{a \in A} \sum_{t=1}^T X_a^{t,n} C^t [\mathbb{E}_{\zeta_e^t}(E_a^{t,n}, \zeta_e^t)]$$

subject to

LS and contiguous constraint

$$C_2 : \overbrace{\sum_{t=t_s^n}^{t_s^n+t_a^n} X_a^{t,n} [\mathbb{E}_{\zeta_h^t}(\zeta_e^t H_a^{t,n})] L^t} = t_a^n, \quad \forall a, n, t$$

$$C_3 : \overbrace{\sum_{t=t_s^n}^{t_s^n+t_a^n} X_a^{t,n} [\mathbb{E}_{\zeta_h^t}(\zeta_e^t H_a^{t,n})] L^t \lambda_a^{t,n}} = t_a^n, \quad \forall a, n, t$$

CP, LS, HIF and Contiguous Constraint

(15)

4.1.2. Robust Optimization Model

Robust optimization model is shown in Equation (16), where ξ_e^t and ξ_h^t are the uncertainty factors. Constraint C_1 is not effected by the uncertainty factor, hence it will remain same as Equation (14). Constraints C_2 and C_3 are effected by the uncertainty factor and are shown in Equation (16). Equality symbol in C_2 and C_3 are replaced by inequality symbol which shows that worst case scenario are satisfied.

$$\begin{aligned}
 & \min_{X_a^{t,n} \in \{0,1\}, \forall a,n} \sum_{n=1}^N \sum_{a \in A} \sum_{t=1}^T \left(\max_{\xi_e^t, \xi_h^t \in \mathcal{U}} X_a^{t,n} C^t(\xi_e^t E_a^{t,n}) \right) \\
 & \text{subject to} \\
 & \quad \underbrace{\text{LS and contiguous constraint}} \\
 & C_2 : \sum_{t=t_s^n}^{t_s^n+t_a^n} X_a^{t,n} (\xi_h^t H_a^{t,n}) L^t \geq t_a^n, \quad \forall a, n, t, \xi_h^t \in \mathcal{U} \\
 & C_3 : \sum_{t=t_s^n}^{t_s^n+t_a^n} X_a^{t,n} (\xi_h^t H_a^{t,n}) L^t \lambda_a^{t,n} \geq t_a^n, \quad \forall a, n, t, \xi_h^t \in \mathcal{U} \\
 & \quad \underbrace{\text{CP, LS, HIF and Contiguous Constraint}}
 \end{aligned} \tag{16}$$

Transforming min–max problem in (16) into minimization problem gives Equation (17)

$$\begin{aligned}
 & \min_{\Theta_a^{t,n}, X_a^{t,n} \in \{0,1\}, \forall a,n} \sum_{n=1}^N \sum_{a \in A} \sum_{t=1}^T \Theta_a^{t,n} \\
 & \text{subject to} \\
 & \quad \underbrace{\text{Contiguous Constraint}} \\
 & C_1 : \sum_{t=t_s^n}^{t_s^n+t_a^n} X_a^{t,n} = t_a^n, \quad \forall a, n, t \\
 & \quad \underbrace{\text{LS and contiguous constraint}} \\
 & C_2 : \sum_{t=t_s^n}^{t_s^n+t_a^n} X_a^{t,n} (\xi_h^t H_a^{t,n}) L^t \geq t_a^n, \quad \forall a, n, t, \xi_h^t \in \mathcal{U} \\
 & C_3 : \sum_{t=t_s^n}^{t_s^n+t_a^n} X_a^{t,n} (\xi_h^t H_a^{t,n}) L^t \lambda_a^{t,n} \geq t_a^n, \quad \forall a, n, t, \xi_h^t \in \mathcal{U} \\
 & \quad \underbrace{\text{CP, LS, HIF and Contiguous Constraint}} \\
 & C_4 : X_a^{t,n} C^t(\xi_e^t E_a^{t,n}) \leq \Theta_a^{t,n}, \quad \forall a, n, t, \xi_e^t, \xi_h^t \in \mathcal{U}
 \end{aligned} \tag{17}$$

4.1.3. Distributionally Robust Optimization Model

Distributionally robust model for the system is shown in (18) where P_e^t and P_h^t are the probability distributions of uncertainty in total energy consumed $E_a^{t,n}$ and human interaction factor $H_a^{t,n}$. D is the ambiguity set and $\mathbb{E}_{\xi_e^t}(\xi_e^t E_a^{t,n})$ is the expected value of energy consumed at time T . Constraints C_2 and C_3 are effected by the uncertainty factors and are shown in Equation (18). Equality symbol in C_2 and C_3 are replaced by the inequality symbol which shows that worst case scenario are satisfied.

$$\begin{aligned}
& \min_{X_a^{t,n} \in \{0,1\} \forall a,n} \sum_{n=1}^N \sum_{a \in A} \sum_{t=1}^T \max_{(P_e^t, P_h^t) \in \mathcal{D}} X_a^{t,n} C^t [\mathbb{E}_{\zeta^t} (\zeta_e^t E_a^{t,n})] \\
& \text{subject to} \\
& \quad \text{LS and contiguous constraint} \\
& C_2 : \sum_{t=t_s^n}^{t_s^n+t_a^n} X_a^{t,n} [\mathbb{E}_{\zeta_h^t} (\zeta_h^t H_a^{t,n})] L^t \geq t_a^n, \quad \forall a, n, t, \zeta_h^t \in \mathcal{U} \\
& C_3 : \sum_{t=t_s^n}^{t_s^n+t_a^n} X_a^{t,n} [\mathbb{E}_{\zeta_e^t} (\zeta_e^t H_a^{t,n})] L^t \lambda_a^{t,n} \geq t_a^n, \quad \forall a, n, t, \zeta_e^t \in \mathcal{U} \\
& \quad \text{CP, LS, HIF and Contiguous Constraint}
\end{aligned} \tag{18}$$

4.1.4. Chance Constrained Optimization Model

Chance constrained optimization model is shown in Equation (19), where ζ_e^t and ζ_h^t are uncertainty factor in total energy consumed and human interaction factors. The confidence level for constraints C_2 and C_3 are γ_1 and γ_2 , respectively, where $\gamma_1, \gamma_2 \in [0, 1]$.

$$\begin{aligned}
& \min_{X_a^{t,n} \in \{0,1\} \forall a,n} \sum_{n=1}^N \sum_{a \in A} \sum_{t=1}^T X_a^{t,n} C^t (\zeta_e^t E_a^{t,n}) \\
& \text{subject to} \\
& \quad \text{LS and contiguous constraint} \\
& C_2 : \mathbb{P} \left(\sum_{t=t_s^n}^{t_s^n+t_a^n} X_a^{t,n} (\zeta_h^t H_a^{t,n}) L^t \geq t_a^n \right) \leq \gamma_1, \quad \forall a, n, t \\
& C_3 : \mathbb{P} \left(\sum_{t=t_s^n}^{t_s^n+t_a^n} X_a^{t,n} (\zeta_e^t H_a^{t,n}) L^t \lambda_a^{t,n} \geq t_a^n \right) \leq \gamma_2, \quad \forall a, n, t \\
& \quad \text{CP, LS, HIF and Contiguous Constraint}
\end{aligned} \tag{19}$$

4.2. Scenario 2: GHG Emission Control Microgrid

Authors in [151] have considered a system as shown in Figure 8. The deterministic mathematical model is developed for unified demand side management, as shown in Equation (20). The objective function is to minimize overall cost of the system including electricity consumption cost $f_c(E_{n,a}^t)$ and carbon dioxide emission cost $f_{CO_2}(E_{n,a}^t)$. The parameter $\alpha_1 \in [0, 1]$ helps in selecting the priorities setting for job operation. Various system's constraints are operation time constraints, appliances continuous operation constraint, consumer preference constraint, human interaction constraint and load shedding constraint, peak clipping, and appliances priority constraint. Let the system has uncertainty in overall energy consumption $E_{n,a}^{k,t}$ and is represented by ζ . Various probabilistic models are formulated to show the impact of uncertainty on the deterministic mathematical model of the system.

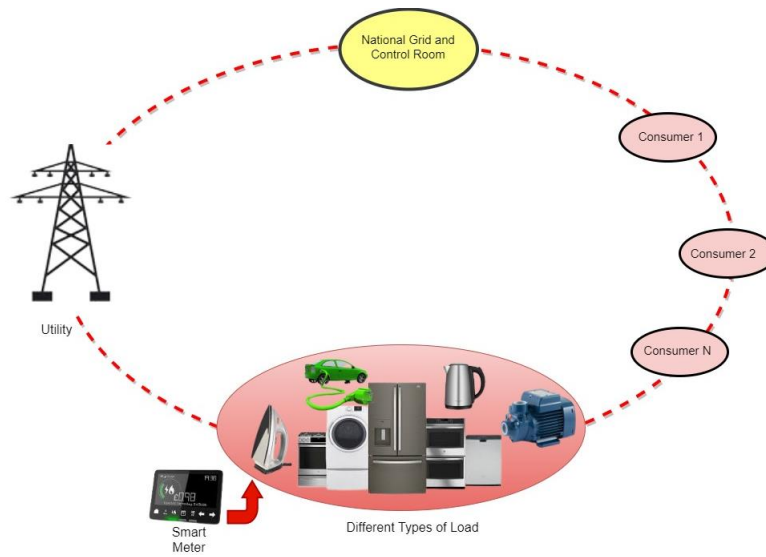


Figure 8. Residential compound architecture.

Deterministic Problem

$$\min_{X_{n,a}^{k,t} \in \{0,1\} \forall k,n,t,a} \sum_{k=1}^K \sum_{n=1}^N \sum_{a \in A_n} \sum_{t=1}^T \left[\overbrace{\left(\alpha_1 \underbrace{f_c(E_{n,a}^{k,t})}_{\text{Electricity cost function}} + (1 - \alpha_1) \underbrace{f_{CO_2}(E_{n,a}^{k,t})}_{\text{Emission cost function}} \right)}^{\text{Total Cost}} X_{n,a}^{k,t} \right] \tag{20}$$

subject to:

$$C1 : \underbrace{\sum_{n=1}^N \sum_{a \in A_n} X_{n,a}^{k,t} E_{n,a}^{k,t}}_{\text{Peak clipping constraint}} \leq \alpha_4 \gamma^{k,t}, \quad \forall k, t$$

Operation time constraint

$$C2 : \sum_{t=1}^T X_{n,a}^{k,t} = t_a^k, \quad \forall k, n, a$$

$$C3 : \sum_{t=ts_{n,a}^k}^{ts_{n,a}^k + t_{n,a}^k - 1} X_{n,a}^{k,t} \geq t_a^k \alpha_5, \quad \forall k, n, a$$

Appliances time continuity constraint

$$C4 : X_{n,a}^{k,t} = X_{n,a}^{k,t} (\lambda_{n,a}^{k,t})^{\alpha_6} (H_{n,a}^{k,t})^{\alpha_3} (L^t)^{\alpha_2}, \quad \forall k, n, a, t$$

Consumer preferences, human interaction and load shedding factors consideration constraint

$$C5 : \sum_{a \in \beta_i} X_{n,a}^{k,t} \leq 1, \quad \forall \text{shiftable appliances}, k, t, \{i = 1, 2, 3 \dots I\}$$

Appliances priorities constraint

4.2.1. Stochastic Optimization Model

Stochastic model for the system could be witnessed by Equation (21), where $\alpha_1 \mathbb{E}_{\xi} f_c(E_{n,a}^{k,t})$ is the expected value of electricity cost and $(1 - \alpha_1) \mathbb{E}_{\xi} f_{CO_2}(E_{n,a}^{k,t})$ is the expected value of CO₂ emission cost. Furthermore, the Constraints C₂ – C₅ are not affected by considering the uncertainty hence they will remain the same as given in Equation (20). In Constraint C₁ the expected value of energy consumption ($\mathbb{E}_{\xi}(\xi E_{n,a}^t)$) at time T is less than certain peak value.

Stochastic Problem

$$\min_{X_{n,a}^{k,t} \in \{0,1\} \forall k,n,t,a} \sum_{k=1}^K \sum_{n=1}^N \sum_{a \in A_n} \sum_{t=1}^T \left[\overbrace{\left(\alpha_1 \underbrace{\mathbb{E}_{\xi} f_c(E_{n,a}^{k,t}, \xi)}_{\text{Electricity cost function}} + (1 - \alpha_1) \underbrace{\mathbb{E}_{\xi} f_{CO_2}(E_{n,a}^t, \xi)}_{\text{Emission cost function}} \right)}^{\text{Total Cost}} X_{n,a}^{k,t} \right]. \quad (21)$$

subject to:

$$C1 : \underbrace{\sum_{n=1}^N \sum_{a \in A_n} X_{n,a}^{k,t} (\mathbb{E}_{\xi}(\xi E_{n,a}^t))}_{\text{Peak clipping constraint}} \leq \alpha_4 \gamma^{k,t}, \quad \forall k, t$$

4.2.2. Robust Optimization Model

In the robust minimization model, which is shown in Equation (22), where the uncertainty ξ in total energy consumed $E_{n,a}^{k,t}$ belongs to an uncertainty set \mathcal{U} . $\alpha_1 f_c(\xi E_{n,a}^{k,t})$ shows the impact of uncertainty on total electricity cost where $(1 - \alpha_1) f_{CO_2}(\xi E_{n,a}^{k,t})$ shows the impact of uncertainty on carbon emission cost. The constraint C₁ is effected by the uncertainty ξ which shows that maximum energy consumption having uncertainty should be less than a certain peak value. Constraints C₂ to C₅ will remain same as in Equation (20). The problem in Equation (22) is a maximization problem that can be converted into a minimization problem as shown in Equation (23).

Robust Problem

$$\min_{X_{n,a}^{k,t} \in \{0,1\} \forall k,n,t,a} \sum_{k=1}^K \sum_{n=1}^N \sum_{a \in A_n} \sum_{t=1}^T \left(\max_{\xi \in \mathcal{U}} \left(\alpha_1 f_c(\xi E_{n,a}^{k,t}) + (1 - \alpha_1) f_{CO_2}(\xi E_{n,a}^{k,t}) \right) X_{n,a}^{k,t} \right). \quad (22)$$

subject to:

$$C1 : \underbrace{\sum_{n=1}^N \sum_{a \in A_n} X_{n,a}^{k,t} (\xi E_{n,a}^{k,t})}_{\text{Peak clipping constraint}} \leq \alpha_4 \gamma^{k,t}, \quad \forall k, t, \xi \in \mathcal{U}$$

Robust Problem 1

$$\min_{W_{n,a}^{k,t}, X_{n,a}^{k,t} \in \{0,1\} \forall k,n,t,a} \sum_{k=1}^K \sum_{n=1}^N \sum_{a \in A_n} \sum_{t=1}^T W_{n,a}^{k,t}. \quad (23)$$

subject to:

$$C1 : \underbrace{\sum_{n=1}^N \sum_{a \in A_n} X_{n,a}^{k,t}(\xi E_{n,a}^{k,t})}_{\text{Peak clipping constraint}} \leq \alpha_4 \gamma^{k,t}, \quad \forall k, t, \xi \in \mathcal{U}$$

$$C2 : \left(\alpha_1 f_c(\xi E_{n,a}^{k,t}) + (1 - \alpha_1) f_{CO_2}(\xi E_{n,a}^{k,t}) X_{n,a}^{k,t} \right) \leq W_{n,a}^{k,t} \quad \forall k, t, \xi \in \mathcal{U}$$

4.2.3. Distributionally Robust Model

Distributionally robust optimization model is shown in Equation (24), where P_{ξ} shows the probability distributions of uncertainty factor ξ that belongs to an ambiguity set D . $\alpha_1 f_c \mathbb{E}_{\xi}[(\xi E_{n,a}^{k,t})]$ is the expected value of uncertain electricity cost, where $(1 - \alpha_1) f_{CO_2} \mathbb{E}_{\xi}[(\xi E_{n,a}^{k,t})]$ is the expected value of uncertain CO₂ emission cost. Constraint C₂ to C₅ will remain same as shown in Equation (20) while C₁ shows the expected of energy consumption is less than a certain peak value.

Distributionally Robust Problem

$$\min_{X_{n,a}^{k,t} \in \{0,1\} \forall k,n,t,a} \sum_{k=1}^K \sum_{n=1}^N \sum_{a \in A_n} \sum_{t=1}^T \left(\max_{P_{\xi} \in D} \left(\alpha_1 f_c \mathbb{E}_{\xi}[(\xi E_{n,a}^{k,t})] + (1 - \alpha_1) f_{CO_2} \mathbb{E}_{\xi}[(\xi E_{n,a}^{k,t})] \right) X_{n,a}^{k,t} \right). \quad (24)$$

subject to:

$$C1 : \underbrace{\sum_{n=1}^N \sum_{a \in A_n} X_{n,a}^{k,t} \mathbb{E}_{\xi}(\xi E_{n,a}^{k,t})}_{\text{Peak clipping constraint}} \leq \alpha_4 \gamma^{k,t}, \quad \forall k, t, \xi \in \mathcal{U}$$

4.2.4. Chance Constrained Optimization Model

The Chance constrained model is shown in Equation (25), where the only affected constraint is C₁. It shows that probability of violating peak clipping limit is less than a certain predefined value, i.e., α . However, Constraint C₂ – C₅ will remain the same as given by Equation (20).

Chance Constrained Problem

$$\min_{X_{n,a}^{k,t} \in \{0,1\} \forall k,n,t,a} \sum_{k=1}^K \sum_{n=1}^N \sum_{a \in A_n} \sum_{t=1}^T \left[\overbrace{\left(\alpha_1 \underbrace{f_c(E_{n,a}^{k,t}, \xi)}_{\text{Electricity cost function}} + (1 - \alpha_1) \underbrace{f_{CO_2}(E_{n,a}^{k,t}, \xi)}_{\text{Emission cost function}} \right) X_{n,a}^{k,t}}^{\text{Total Cost}} \right]. \quad (25)$$

subject to:

$$C1 : \mathbb{P} \left(\underbrace{\sum_{n=1}^N \sum_{a \in A_n} X_{n,a}^{k,t}(\xi E_{n,a}^{k,t})}_{\text{Peak clipping constraint}} \leq \alpha_4 \gamma^{k,t} \right) \leq \alpha, \quad \forall k, t$$

4.3. Scenario 3: Energy Trading Model for Microgrid System

In Figure 9, the network of a microgrid is shown where different sources of generation are considered, e.g., PV cells, small wind turbines and utility. Furthermore, Figure 9

shows that microgrids are connected to each other and with utility. Based on defined tariff, the microgrids can exchange energy with each other as well as with utility. Cost per unit of self-generation is usually low as compared to energy procure from utility and from other microgrids. The system model shown in Figure 9 has a total of V microgrids connected with each other and with utility. N is the total number of consumers on each microgrid, and each consumer has a set of shiftable and non-shiftable appliances. The appliances are operated in such a way so that the self-generation of microgrid v can fulfill the energy demand; if the energy generated by microgrid v is not enough to meet its energy demand than microgrid v will procure energy from other microgrids. If the other microgrids do not have extra energy to sell it to microgrid v than they will procure energy from the utility.

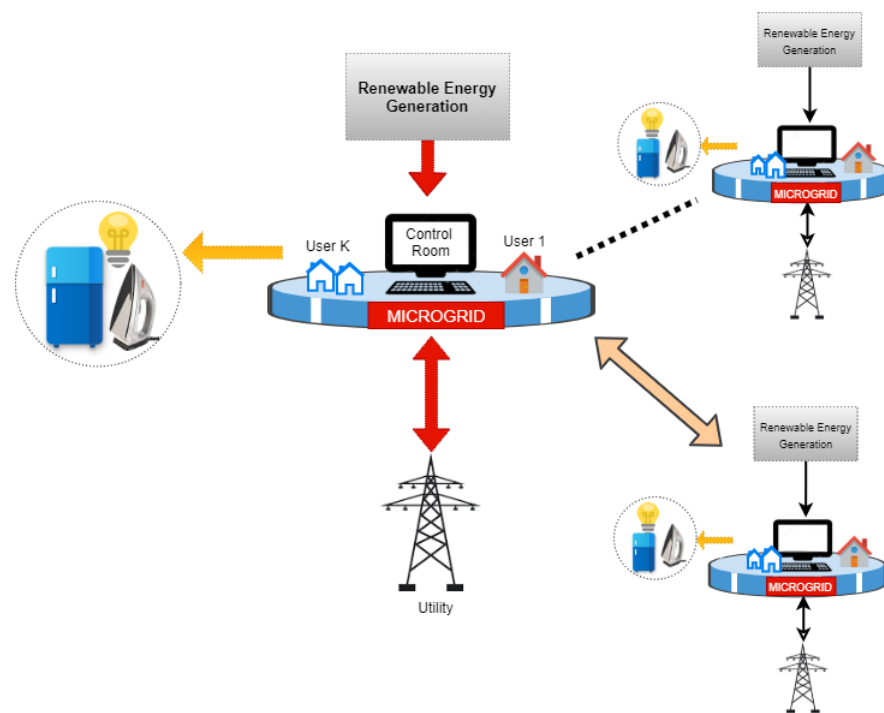


Figure 9. System model for scenarios 3 and 4.

The deterministic mathematical model for the energy trading module in the microgrid system as shown in Equation (26), where the objective function minimizes the overall cost of the system as well as to satisfy various constraints. Positive sign with the decision variables shows that the energy is either generated by the microgrid itself or it sells the extra energy to other microgrids or utility, where negative signs represents that the energy is purchased from the utility or from other microgrids. Peak generation constraint defines the maximum generating capacity of the microgrid. Maximum purchase of energy from utility or from other microgrids are represented by peak purchase constraint. Various other constraints are peak generation constraint, peak purchase from other microgrids and utility, appliances priority constraint, no self sell/purchase constraint, either to sell or purchase constraint. Let the energy generated by microgrid v has the uncertainty ζ_g . To model the impact of uncertainty factor ζ_g over the system, various probabilistic models are shown as follows.

Deterministic Problem

$$\begin{aligned}
 \min_{\substack{E_v^{t,u}, E_n^{t,g}, E_{v,u}^t, \\ E_{w,v}^t, E_{v,w}^t, X_{v,a}^{t,n} \in \{0,1\} \\ \forall v,n,t,a}} & \sum_{t=1}^T \sum_{v=1}^V \left(C(E_v^{t,u}) + C(E_v^{t,g}) - C(E_{v,u}^t) + \sum_{w=1}^V (C(E_{w,v}^t) - C(E_{v,w}^t)) \right) \\
 \text{S.t} & \quad \text{Peak Generation Constraint} \\
 & \quad C1 : \overbrace{0 \leq E_v^{t,g} \leq E_v^{t,g,max}} \quad \forall v, t \\
 & \quad \text{Peak Purchase from Microgrids} \\
 & \quad C2 : \overbrace{0 \leq E_{w,v}^t \leq E_{w,v}^{t,max}} \quad \forall v, w, t \\
 & \quad \text{Peak Purchase from Utility} \\
 & \quad C3 : \overbrace{0 \leq E_v^{t,u} \leq E_v^{t,u,max}} \quad \forall v, t \\
 & \quad \text{Appliances Priority Constraint} \\
 & \quad C4 : \overbrace{\sum_{a \in \beta_1} X_{v,a}^{t_1,n} + \sum_{a \in \beta_2} X_{v,a}^{t_2,n} \leq 1} \quad \forall \{t_1 < t_2\}, n, v \\
 & \quad \text{No Self Sell/Purchase} \\
 & \quad C5 : \overbrace{E_{v,v}^t = 0} \quad \forall v, t \\
 & \quad \text{Either Sell or Purchase Constraint} \\
 & \quad C6 : \overbrace{E_{v,w}^t \times E_{w,v}^t = 0} \quad \forall t, v, w
 \end{aligned} \tag{26}$$

4.3.1. Stochastic Optimization Model

The stochastic optimization model could be represented such as in Equation (27), where the $C[\mathbb{E}_{\xi_g}(E_v^{t,g}, \xi_g)]$ is the expected value of the energy generated by microgrid v . The only constraint that is affected by considering uncertainty is C_1 , while the rest of the constraints will remain the same as Equation (26) witnesses it. The factor $\mathbb{E}_{\xi_g}(\xi_g E_v^{t,g})$ in constraint C_1 shows that expected value of uncertain generation supposed to be less than certain defined value.

Stochastic Problem

$$\begin{aligned}
 \min_{\substack{E_v^{t,u}, E_n^{t,g}, E_{v,u}^t, \\ E_{w,v}^t, E_{v,w}^t, X_{v,a}^{t,n} \in \{0,1\} \\ \forall v,n,t,a}} & \sum_{t=1}^T \sum_{v=1}^V \left(C(E_v^{t,u}) + C[\mathbb{E}_{\xi_g}(E_v^{t,g}, \xi_g)] - C(E_{v,u}^t) + \sum_{w=1}^V (C(E_{w,v}^t) - C(E_{v,w}^t)) \right) \\
 \text{S.t} & \quad \text{Peak Generation Constraint} \\
 & \quad C1 : \overbrace{0 \leq \mathbb{E}_{\xi_g}(\xi_g E_v^{t,g}) \leq E_v^{t,g,max}} \quad \forall v, t
 \end{aligned} \tag{27}$$

4.3.2. Robust Optimization Model

Robust optimization model is shown in Equation (28), where $C(\xi_g E_v^{t,g})$ show the energy generated by microgrid v having uncertainty ξ . The constraint C_1 which is effected by considering uncertainty, shows that the energy generated by microgrid v at time t varies in an uncertain manner but it should be less than a certain maximum value. The min–max problem of Equation (28) can be transformed into minimization problem as shown in Equation (29).

Robust Problem

$$\begin{aligned}
 & \min_{\substack{E_v^{t,\mu}, E_n^{t,g}, E_{v,\mu}^t, \\ E_{w,v}^t, E_{v,w}^t, X_{v,a}^{t,n} \in \{0,1\} \\ \forall v,n,t,a}} & \sum_{t=1}^T \sum_{v=1}^V \max_{\xi_g \in \mathcal{U}} \left(C(E_v^{t,\mu}) + C(\xi_g E_v^{t,g}) - C(E_{v,\mu}^t) + \sum_{w=1}^V (C(E_{w,v}^t) - C(E_{v,w}^t)) \right) \\
 \text{s.t.} & \quad \overbrace{C1 : 0 \leq \xi_g E_v^{t,g} \leq E_v^{t,g,max}}^{\text{Peak Generation Constraint}} \quad \forall v,t, \xi_g \in \mathcal{U}
 \end{aligned} \tag{28}$$

Robust Problem 1

$$\begin{aligned}
 & \min_{\substack{Z_v^t, X_{v,a}^{t,n} \in \{0,1\} \\ \forall v,n,t,a}} & \sum_{t=1}^T \sum_{v=1}^V Z_v^t \\
 \text{s.t.} & \quad \overbrace{C1 : 0 \leq \xi_g E_v^{t,g} \leq E_v^{t,g,max}}^{\text{Peak Generation Constraint}} \quad \forall v,t, \xi_g \in \mathcal{U} \\
 & \quad C2 : \left(C(E_v^{t,\mu}) + C(\xi_g E_v^{t,g}) - C(E_{v,\mu}^t) + \sum_{w=1}^V (C(E_{w,v}^t) - C(E_{v,w}^t)) \right) \leq Z_v^t \\
 & \quad \forall v,t, \xi_g \in \mathcal{U}
 \end{aligned} \tag{29}$$

4.3.3. Distributionally Robust Optimization

Distributionally robust model is shown in Equation (30), where the P_g indicates the probability distributions of uncertainty in energy generated by microgrid v . $\mathbb{E}_g(\xi_g E_v^{t,g})$ is the expected value of the energy generated by microgrid v at time t . The constraint C_1 is effected by considering the uncertainty while the constraints C_2 to C_6 will remain same as shown by Equation (26).

Distributionally Robust Problem

$$\begin{aligned}
 & \min_{\substack{E_v^{t,\mu}, E_n^{t,g}, E_{v,\mu}^t, \\ E_{w,v}^t, E_{v,w}^t, X_{v,a}^{t,n} \in \{0,1\} \\ \forall v,n,t,a}} & \sum_{t=1}^T \sum_{v=1}^V \max_{P_g \in \mathcal{D}} \left(C(E_v^{t,\mu}) + C\mathbb{E}_g(\xi_g E_v^{t,g}) - C(E_{v,\mu}^t) + \sum_{w=1}^V (C(E_{w,v}^t) - C(E_{v,w}^t)) \right) \\
 \text{s.t.} & \quad \overbrace{C1 : 0 \leq \mathbb{E}_g(\xi_g E_v^{t,g}) \leq E_v^{t,g,max}}^{\text{Peak Generation Constraint}} \quad \forall v,t, \xi_g \in \mathcal{U}
 \end{aligned} \tag{30}$$

4.3.4. Chance Constrained Optimization Model

The chance constrained model is shown in Equation (31), where constraint C_1 shows that the probability of violating the peak generation limit is less than a certain predefined value. Constrain from $C_2 - C_6$ will remain same as shown in Equation (26).

Chance Constrained Problem

$$\begin{aligned}
 & \min_{\substack{E_v^{t,\mu}, E_n^{t,g}, E_{v,\mu}^t, \\ E_{w,v}^t, E_{v,w}^t, X_{v,a}^{t,n} \in \{0,1\} \\ \forall v,n,t,a}} & \sum_{t=1}^T \sum_{v=1}^V \left(C(E_v^{t,\mu}) + C(E_v^{t,g}, \xi_g) - C(E_{v,\mu}^t) + \sum_{w=1}^V (C(E_{w,v}^t) - C(E_{v,w}^t)) \right) \\
 \text{s.t.} & \quad \overbrace{C1 : \mathbb{P}\left(0 \leq \xi_g E_v^{t,g} \leq E_v^{t,g,max}\right) \leq \zeta}^{\text{Peak Generation Constraint}} \quad \forall v,t
 \end{aligned} \tag{31}$$

4.4. Scenario 4: Joint Energy Management and Trading for Microgrid System

In this section, energy management and energy trading models are collectively considered, where the deterministic model of the system is formulated and described in [152]. The system model for joint energy management and trading is shown in Figure 9. Various energy management constraints such as energy balance constraint, peak clipping constraint, and operation time constraint have been considered in addition to energy trading constraints that are mentioned in scenario 3. Let the applied model in [152] have uncertainty in various parameters such as energy generated ($E_v^{t,g}$) by microgrid is v and base-load (B_v^t) on microgrid v . The uncertainty in $E_v^{t,g}$ and B_v^t are represented as ζ_g and ζ_l , respectively. The objective function and various constraints such as energy balance constraint, peak clipping constraint and peak generation constraint are effected by considering uncertainty in energy generation and base-load. The remaining constraints of the system will remain the same as shown in [152]. To show the impact of uncertainty, various probabilistic optimization schemes are applied as follows.

Deterministic Problem

$$\begin{aligned}
 \min_{\substack{E_v^{t,\mu}, E_v^{t,g}, E_v^t, \\ E_{w,v}^t, E_{v,w}^t, X_{v,a}^{t,n} \in \{0,1\} \\ \forall v,n,t,a}} & \sum_{t=1}^T \sum_{v=1}^V \left(C(E_v^{t,\mu}) + C(E_v^{t,g}) - C(E_{v,\mu}^t) + \sum_{w=1}^V (C(E_{w,v}^t) - C(E_{v,w}^t)) \right) \quad (32) \\
 \text{S.t.} & \quad \overbrace{C1: E_v^{t,\mu} + E_v^{t,g} + \sum_{w=1}^V E_{w,v}^t = \sum_{w=1}^V E_{v,w}^t + E_{v,\mu}^t + B_v^t + \sum_{a \in A^n} X_{v,a}^{t,n} L_{v,a}^{t,n}}^{\text{Energy Balance Constraint}} \quad \forall v, w, n, t \\
 & \quad \overbrace{C2: \sum_{t=t_a^{s,n}}^{t_a^{s,n} + t_a^n} (X_{v,a}^{t,n} H_{v,a}^{t,n} L_{v,a}^{t,n} L_{v,a}^t \lambda_{v,a}^{t,n} + B_v^t) \leq \gamma^t}^{\text{Peak Clipping Constraint}} \quad \forall n, a, v \\
 & \quad \overbrace{C3: \sum_{t=t_a^{s,n}}^{t_a^{s,n} + t_a^n} X_{v,a}^{t,n} H_{v,a}^{t,n} L_{v,a}^t \lambda_{v,a}^{t,n} = t_a^n}^{\text{Operation Time Constraint}} \quad \forall n, a, v \\
 & \quad \overbrace{C4: 0 \leq E_v^{t,g} \leq E_v^{t,g,max}}^{\text{Peak Generation Constraint}} \quad \forall v, t \\
 & \quad \overbrace{C5: 0 \leq E_{w,v}^t \leq E_{w,v}^{t,max}}^{\text{Peak Purchase from Microgrids}} \quad \forall v, w, t \\
 & \quad \overbrace{C6: 0 \leq E_v^{t,\mu} \leq E_v^{t,\mu,max}}^{\text{Peak Purchase from Utility}} \quad \forall v, t \\
 & \quad \overbrace{C7: \sum_{a \in \beta_1} X_{v,a}^{t_1,n} + \sum_{a \in \beta_2} X_{v,a}^{t_2,n} \leq 1}^{\text{Appliances Priority Constraint}} \quad \forall \{t_1 < t_2\}, n, v \\
 & \quad \overbrace{C8: E_{v,v}^t = 0}^{\text{No Self Sell/Purchase}} \quad \forall v, t \\
 & \quad \overbrace{C9: E_{v,w}^t \times E_{w,v}^t = 0}^{\text{Either Sell or Purchase Constraint}} \quad \forall t, v, w
 \end{aligned}$$

4.4.1. Stochastic Optimization Model

In stochastic optimization, the expected value of uncertain parameters is considered. The stochastic mathematical model of the considered system is given by Equation (33). Where, in Equation (33) the $\mathbb{E}_{\zeta_g} [C(E_v^{t,g}, \zeta_g)]$ shows the expected value of total cost of the energy generated by microgrid v , where in constrains $\mathbb{E}_{\zeta_g} (\zeta_g E_v^{t,g})$ is the expected value of

the energy generated by microgrid v and $\mathbb{E}_{\zeta_l}(\zeta_l B_v^t)$ is the expected value of the base load on the microgrid v .

Stochastic Problem

$$\min_{\substack{E_v^{t,\mu}, E_n^{t,g}, E_{v,\mu}^t, \\ E_{w,v}^t, E_{v,w}^t, X_{v,a}^{t,n} \in \{0,1\} \\ \forall v,n,t,a}} \sum_{t=1}^T \sum_{v=1}^V \left(C(E_v^{t,\mu}) + \mathbb{E}_{\zeta_g} [C(E_v^{t,g}, \zeta_g)] - C(E_{v,\mu}^t) + \sum_{w=1}^V (C(E_{w,v}^t) - C(E_{v,w}^t)) \right) \tag{33}$$

Energy Balance Constraint

$$S.t \quad C1 : E_v^{t,\mu} + \mathbb{E}_{\zeta_g} [\zeta_g E_v^{t,g}] + \sum_{w=1}^V E_{w,v}^t = \sum_{w=1}^V E_{v,w}^t + E_{v,\mu}^t + \mathbb{E}_{\zeta_l} [\zeta_l B_v^t] + \sum_{a \in A^n} X_{v,a}^{t,n} L_{v,a}^{t,n} \quad \forall v, w, n, t$$

Peak Clipping Constraint

$$C2 : \sum_{t=t_a^{s,n}}^{t_a^{e,n} + t_a^n} (X_{v,a}^{t,n} H_{v,a}^{t,n} L_{v,a}^{t,n} L_v^t \lambda_{v,a}^{t,n} + \mathbb{E}_{\zeta_l} [\zeta_l B_v^t]) \leq \gamma^t \quad \forall n, a, v$$

Peak Generation Constraint

$$C4 : 0 \leq \mathbb{E}_{\zeta_g} (\zeta_g E_v^{t,g}) \leq E_v^{t,g,max} \quad \forall v, t$$

4.4.2. Robust Optimization Model

Robust optimization considers a worst-case scenario for the system. It is assumed that the uncertainty factor ζ_g can take values 0.9 – 1.0 and the uncertainty factor ζ_l can varies from 1.0 – 1.2. The robust mathematical model for the system considered is shown in Equation (34). Equality constraint C₁ is changed to inequality constraint, which ensures that the load balancing constraint should be satisfied in the worst case. The problem in Equation (34) is a min–max problem that can be converted into a minimization problem, as shown in Equation (35).

Robust Problem

$$\min_{\substack{E_v^{t,\mu}, E_n^{t,g}, E_{v,\mu}^t, \\ E_{w,v}^t, E_{v,w}^t, X_{v,a}^{t,n} \in \{0,1\} \\ \forall v,n,t,a}} \sum_{t=1}^T \sum_{v=1}^V \max_{(\zeta_g, \zeta_l) \in \mathcal{U}} \left(C(E_v^{t,\mu}) + C(\zeta_g E_v^{t,g}) - C(E_{v,\mu}^t) + \sum_{w=1}^V (C(E_{w,v}^t) - C(E_{v,w}^t)) \right) \tag{34}$$

Energy Balance Constraint

$$S.t \quad C1 : E_v^{t,\mu} + \zeta_g E_v^{t,g} + \sum_{w=1}^V E_{w,v}^t \geq \sum_{w=1}^V E_{v,w}^t + E_{v,\mu}^t + \zeta_l B_v^t + \sum_{a \in A^n} X_{v,a}^{t,n} L_{v,a}^{t,n} \quad \forall v, w, n, t, (\zeta_g, \zeta_l) \in \mathcal{U}$$

Peak Clipping Constraint

$$C2 : \sum_{t=t_a^{s,n}}^{t_a^{e,n} + t_a^n} (X_{v,a}^{t,n} H_{v,a}^{t,n} L_{v,a}^{t,n} L_v^t \lambda_{v,a}^{t,n} + \zeta_l B_v^t) \leq \gamma^t \quad \forall n, a, v, \zeta_l \in \mathcal{U}$$

Peak Generation Constraint

$$C4 : 0 \leq \zeta_g E_v^{t,g} \leq E_v^{t,g,max} \quad \forall v, t, \zeta_g \in \mathcal{U}$$

$$\begin{aligned}
 & \min_{\substack{Y_v^t, X_{v,a}^{t,n} \in \{0,1\} \\ \forall v,n,t,a}} \sum_{t=1}^T \sum_{v=1}^V Y_v^t & (35) \\
 \text{S.t.} & \quad \overbrace{C1 : E_v^{t,u} + \zeta_g E_v^{t,g} + \sum_{w=1}^V E_{w,v}^t \geq \sum_{w=1}^V E_{v,w}^t + E_{v,u}^t + \zeta_l B_v^t + \sum_{a \in A^n} X_{v,a}^{t,n} L_{v,a}^{t,n}}^{\text{Energy Balance Constraint}} \quad \forall v, w, n, t, (\zeta_g, \zeta_l) \in \mathcal{U} \\
 & \quad \overbrace{C2 : \sum_{t=t_a^{s,n}}^{t_a^{s,n} + t_a^n} (X_{v,a}^{t,n} H_{v,a}^{t,n} L_{v,a}^{t,n} L_v^t \lambda_{v,a}^{t,n} + \zeta_l B_v^t) \leq \gamma^t}^{\text{Peak Clipping Constraint}} \quad \forall n, a, v, \zeta_l \in \mathcal{U} \\
 & \quad \overbrace{C4 : 0 \leq \zeta_g E_v^{t,g} \leq E_v^{t,g,max}}^{\text{Peak Generation Constraint}} \quad \forall v, t, \zeta_g \in \mathcal{U} \\
 & \quad C5 : C(E_v^{t,u}) + C(\zeta_g E_v^{t,g}) - C(E_{v,u}^t) + \sum_{w=1}^V (C(E_{w,v}^t) - C(E_{v,w}^t)) \leq Y_v^t \quad \forall v, w, n, t, (\zeta_g, \zeta_l) \in \mathcal{U}
 \end{aligned}$$

4.4.3. Distributionally Robust Optimization

Mathematical model for distributionally robust optimization is shown in Equation (36), where P_g and P_l are the probability distributions for the uncertain parameters ζ_g and ζ_l , respectively. ζ_g and ζ_l represent uncertainty in energy generated and base load on microgrid v at time t , respectively. $\mathbb{E}_g[E_v^{t,g}, \zeta_g]$ is the expected value of energy generated by microgrid v at time t , where $\mathbb{E}_l[\zeta_l B_v^t]$ is the expected value of base load on microgrid V . Constraints C_1 to C_3 are effected by considering uncertainty while the constraints C_4 to C_5 will remain same as in Equation (32).

Distributionally Robust Problem

$$\begin{aligned}
 & \min_{\substack{E_v^{t,u}, E_v^{t,g}, E_{v,u}^t, \\ E_{w,v}^t, E_{v,w}^t, X_{v,a}^{t,n} \in \{0,1\} \\ \forall v,n,t,a}} \sum_{t=1}^T \sum_{v=1}^V \max_{(P_g, P_l) \in \mathcal{D}} \left(C(E_v^{t,u}) + C(\mathbb{E}_g[E_v^{t,g}, \zeta_g]) - C(E_{v,u}^t) + \sum_{w=1}^V (C(E_{w,v}^t) - C(E_{v,w}^t)) \right) & (36) \\
 \text{S.t.} & \quad \overbrace{C1 : E_v^{t,u} + \mathbb{E}_g[\zeta_g E_v^{t,g}] + \sum_{w=1}^V E_{w,v}^t \geq \sum_{w=1}^V E_{v,w}^t + E_{v,u}^t + \mathbb{E}_l[\zeta_l B_v^t] + \sum_{a \in A^n} X_{v,a}^{t,n} L_{v,a}^{t,n}}^{\text{Energy Balance Constraint}} \quad \forall v, w, n, t, (\zeta_g, \zeta_l) \in \mathcal{U} \\
 & \quad \overbrace{C2 : \sum_{t=t_a^{s,n}}^{t_a^{s,n} + t_a^n} (X_{v,a}^{t,n} H_{v,a}^{t,n} L_{v,a}^{t,n} L_v^t \lambda_{v,a}^{t,n} + \mathbb{E}_l[\zeta_l B_v^t]) \leq \gamma^t}^{\text{Peak Clipping Constraint}} \quad \forall n, a, v, \zeta_l \in \mathcal{U} \\
 & \quad \overbrace{C4 : 0 \leq \mathbb{E}_g[\zeta_g E_v^{t,g}] \leq E_v^{t,g,max}}^{\text{Peak Generation Constraint}} \quad \forall v, t, \zeta_g \in \mathcal{U}
 \end{aligned}$$

4.4.4. Chance Constrained Optimization Model

Mathematically speaking, in chance constrained optimization, the probability of satisfying certain constraints should be above a certain predefined level, improving the confidence level of the solution [153,154]. The mathematical model of chance constrained optimization is available in Equation (37). α and β are the confidence level for energy balance and peak clipping constraints, respectively, where $\alpha, \beta \in [0, 1]$.

Chance Constrained

$$\begin{aligned}
 & \min_{\substack{E_v^{t,\mu}, E_n^{t,g}, E_{v,\mu}^t, \\ E_{w,v}^t, E_{v,w}^t, X_{v,a}^{t,n} \in \{0,1\} \\ \forall v,n,t,a}} \\
 & \sum_{t=1}^T \sum_{v=1}^V \left(C(E_v^{t,\mu}) + C(E_v^{t,g}, \zeta_g) - C(E_{v,\mu}^t) + \sum_{w=1}^V (C(E_{w,v}^t) - C(E_{v,w}^t)) \right) \quad (37)
 \end{aligned}$$

$$\begin{aligned}
 \text{S.t.} \quad & \text{C1: } \mathbb{P} \left(\overbrace{\left(E_v^{t,\mu} + \zeta_g E_v^{t,g} + \sum_{w=1}^V E_{w,v}^t = \sum_{w=1}^V E_{v,w}^t + E_{v,\mu}^t + \zeta_l B_v^t + \sum_{a \in A^n} X_{v,a}^{t,n} L_{v,a}^{t,n} \right)}^{\text{Energy Balance Constraint}} \right) \leq \alpha \quad \forall v, w, n, t \\
 & \text{C2: } \mathbb{P} \left(\overbrace{\left(\sum_{t=t_a^n}^{t_a^n + t_a^n} (X_{v,a}^{t,n} H_{v,a}^{t,n} L_{v,a}^{t,n} L_v^t \lambda_{v,a}^{t,n} + \zeta_l B_v^t \geq \gamma^t) \right)}^{\text{Peak Clipping Constraint}} \right) \leq \beta \quad \forall n, a, v \\
 & \text{C4: } \mathbb{P} \left(\overbrace{\left(0 \leq \zeta_g E_v^{t,g} \leq E_v^{t,g,max} \right)}^{\text{Peak Generation Constraint}} \right) \leq \zeta \quad \forall v, t
 \end{aligned}$$

5. Challenges and Future Research Directions

After a comprehensive survey on the smart power system (having uncertainties in numerous domains), it is recommended to focus on limitations and challenges due to these uncertainties and improve the smart power system's performance. These limitations and challenges arise in various segments of smart power system such as microgrid energy management, home energy management, demand side management, renewable energy resources, energy storage system, unit commitment, economic dispatch and transmission system.

5.1. Microgrid Energy Management

A microgrid is an essential and significant part of a smart power system. It is operated both in grid connected as well as in an isolated mode. Microgrid energy management problems' accuracy is seriously affected by the uncertainties in various factors such as renewable energy resources, plug-in electric vehicle and load, etc. Research papers studied have considered uncertainties in few aspects at a time. Hence, it open doors for researchers to model microgrid energy-management problems in a comprehensive way, where uncertainties in all factors affecting microgrid operation can be considered. Furthermore, a model can be developed where the various techniques can be applied in a combined way. The impact of environmental conditions can also be considered in microgrid energy management.

5.2. Demand Side Management

Demand side management plays a critical role in the energy optimization of a smart power system. Demand-side management mainly deals with the consumers' end; hence, the uncertainties at the consumers end have severe impact on the smart power system's performance. Various components that cause uncertainties at consumers' end are manually operated appliances, renewable energy generation, electric vehicle and distributed energy storage devices, inelastic load demand, etc. Therefore, it is an open research direction in the smart power system area to develop a model that can consider the impact of uncertainties caused by the sources mentioned above.

5.3. Integration of Distribution Energy Resources

One of the most significant elements of the smart power system is distributed energy resources. Wind and solar energy are considered prominent example of distributed energy resources. The output power of these sources is seriously affected by the weather condition, which causes uncertainty. Due to the integration of DER with the smart power system, the smart power system's performance is affected by the uncertainties. The various

model used in the literature have considered a single source of uncertainty; hence, it is an open research area to comprehensively develop a model that can completely handle the uncertainty of distributed energy resources. Furthermore, various optimization techniques that can handle uncertainties can be considered in a combined way to improve the model's performance.

5.4. Smart Home

To increase consumers' participation and user comfort, reducing peak demand and consumer electricity bill, it is required to introduce a smart home concept in a smart power system. Various components of a smart home are renewable energy resources, battery storage system and load. Uncertainties are caused by generating resources due to their weather dependent nature. Load have uncertainties either due to its weather dependent behaviour as well as because of the random interaction of human. The impact of these uncertainties on the power system can be developed comprehensively in future research.

5.5. Unit Commitment

Unit commitment is an optimization problem that determines the generating units' operation schedule at certain time intervals with changing loads under different systems' constraints and environmental conditions. Unit commitment plays a vital role in the economic operation of a smart power system. Hence, to minimize the smart power system's operation cost, it is required to consider all factors affecting the unit commitment problems. The complexity of the unit commitment optimization problems increases with the inclusion of uncertainties in the system parameters. Uncertainties in unit commitment problems are due to the integration of renewable energy resources, power flow-ability of a transmission line, forecasted errors in load, unexpected generator outages etc. It can be considered a future research area to develop a unified unit commitment model that can handle all factors having uncertainties.

6. Conclusions

It is a known fact that real time problems of smart power systems have uncertainties and can be handled only by probabilistic optimization. This review paper has been focused on various aspects of probabilistic optimization in smart power system. Various techniques such as stochastic optimization, robust optimization, distributionally robust optimization, and chance constrained optimization have been thoroughly studied in this paper to deal with such type of uncertainties. From the analysis of different concrete study cases, it is observed that stochastic optimization is providing a reliable solution, however its computational cost is high. In worst case condition, robust optimization has a lower computational cost, but it offers a conservative and cautious solution. The solution of distributionally robust optimization is less conservative than robust and has a lower computational cost than stochastic optimization. Chance constrained optimization deals with problems, where finite probability is being violated. These techniques can be further categorized on the basis of the uncertainty involved. Various applications of the mentioned optimization approach in the area of smart power system have been discussed. Future directions of these techniques with reference to smart power system have been summarized. It has been concluded that these techniques must be used in combination to deal with new challenges of smart power system for achieving fruitful outcomes.

Author Contributions: Conceptualization, M.R. and M.N.; methodology, M.N. and S.A.; software, M.R. and I.H.; validation, M.R., I.H. and L.M.-P.; formal analysis, M.R. and M.N.; investigation, S.A. and I.H.; resources, I.H. and M.N.; data curation, S.A. and M.N.; writing—original draft preparation, M.R.; writing—review and editing, S.A. and I.H.; visualization, L.M.-P.; supervision, M.N. and L.M.-P. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Acknowledgments: The authors are extremely thankful to anonymous reviewers for enhancing the quality of this paper via their valuable comments.

Conflicts of Interest: The authors declare no conflict of interest.

Abbreviations

The following abbreviations are used in this manuscript:

MEM	Micro-grid Energy Management
HEM	Home Energy Management
DER	Distributed Energy Management
SDN	Smart Distribution Network
DSM	Demand Side Management
PEV	Plugin Electric Vehicles
ED	Economic Dispatch
UC	Unit Commitment
STG	Smart Thermal Grid
MEO	Micro-grid Economic Operation
RDG	Reconfiguration of Distribution Grid
OPF	Optimal Power Flow
ERD	Energy and Reserve Dispatch
ESS	Energy Storage System
SPDA	Scenario Partition and Decomposition Algorithm
CCDCGP	Chance Constrained dependent chance goal programming
FMEA	Failure-Mode-and Effect analysis
IPEA	Inter-generation Projection Evolutionary Algorithm
IGDT	Information Gap Decision Theory
MPC	Model Predictive Control
FPIM	Fuzzy Prediction Interval Model
SPD	Scenario Partition and Decomposition
BMLM	Big-M Linearization Method
LOP	Lyapunov Optimization Method
CCG	Column-and-Constraint Generation
AM	Analytic Method
LDR	Linear Decision Rule
MH	Math-Heuristic
BD	Benders Decomposition
TOA	Taguchis Orthogonal Array
DD	Dual Decomposition
BB	Branch-and-Bound
LM	Lagrangian Multiplier
QP	Quadratic Programming
MCS	Monte Carlo Simulation
IM	Iterative Method
SAA	Sample Average Approximation
SBM	Scenario Based Method
IPM	Interior Point Methods
DE	Differential Evolution
HABC	Hybrid Artificial Bee Colony

POC	Pareto-optimal cuts
DD	Dual Decomposition
SA	Sensitivity Analysis
SVM	Support Vector Machine
LR	Linear Regression
MDP	Markov Decision Process
SO	Stochastic Optimization
RO	Robust Optimization
CCO	Chance Constrained Optimization
DRO	Distributional Robust Optimization
SA	Solution Algorithms
OF	Objective Function
FRD	Future Research Directions
GEM	Grid Energy Management
TPEM	Two-Point Estimate Method
OPGF	Optimal Power Gas Flow
SGTD	Smart Grid Tariff Design
HE	Heuristic
CC	Chance Constrained
AR	Architecture
TN	Taxonomy

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