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"We learn it [mathematics] at school so one thinks that one will use it …": Learners' beliefs about relevance and importance of learning mathematics

Abstract

In this study, we explore Norwegian learners' beliefs about the relevance and importance of learning mathematics. The data material consists of semi-structured interviews with nineteen lower secondary school learners (13-14 years old). The analyses indicate that the learners believe in the relevance of learning secondary level mathematics though they struggle to give examples of this relevance in their personal lives. Further, learning mathematics is also believed to be important for their lives but the sources of these beliefs are often statements heard from others. The analyses also suggest that the learners seldom reflect, evaluate, or question the relevance and importance of learning mathematics critically. A scarcity of such reflections indicates a tension between learners' beliefs and the curriculum's aim to empower learners to think critically and contribute to the decisions concerning their own mathematics learning process. We discuss the implications of this tension for learners' empowerment and development as critical citizens, who can participate and voice their opinions in discourses about the significance and roles of mathematics education in their personal lives and society.

Keywords: relevance, importance, mathematics education, learners` beliefs, sociopolitical discourse, learners` empowerment

«Vi lærer det [matematikk] på skolen så da tenker du at man har bruk for det …»: Elevenes oppfatninger om relevans og viktighet av å lære matematikk

Sammendrag

I denne studien utforsker vi norske elevers oppfatninger om relevansen til matematikken de lærer og hvor viktig det er for dem å lære matematikk. Datamaterialet består av semistrukturerte intervjuer med nitten ungdomsskoleelever (13-14 år) i Norge. Analysene viser at elevene oppfatter matematikken de lærer som relevant, men også at de har problemer med å gi eksempler på slik relevans for dem selv. Videre svarer elevene at det er viktig å lære matematikk, men når de skal begrunne hvorfor, viser de til uttalelser fra andre. Dataanalysen indikerer videre at elevene sjelden reflekterer over, vurderer, eller stiller kritiske spørsmål ved relevansen og viktigheten av matematikken de lærer. Det kom fram få slike kritiske vurderinger, noe som indikerer et avvik mellom elevenes oppfatninger og formålparagrafens mål om at elevene skal tenke kritisk og medvirke i avgjørelser som angår deres egen opplæring. Vi drøfter betydningen av disse funnene og hva avviket betyr for å myndiggjøre elevene og deres utvikling til å bli kritisk tenkende borgere som kan vurdere og diskutere rollene matematikkopplæringen har i deres egne liv og i samfunnet.

Nøkkelord: relevans, viktighet, matematikkopplæring, elevenes oppfatninger, sosiopolitisk diskurs, elevenes myndiggjøring

Introduction

Learners' beliefs about learning and teaching of mathematics can influence their attitude and motivation to learn mathematics (Grootenboer & Marshman, 2016a; Leder et al., 2002). Therefore, their beliefs about the relevance and importance of learning secondary level mathematics in their present and future lives are investigated thoroughly (see e.g., Grootenboer and Marshman (2016b); Kollosche (2017); Onion (2004) and Pais (2013)). Findings of these studies reveal that learners believe school mathematics to be relevant and important for their lives (Kollosche, 2017; Onion, 2004; Pais, 2013), and have a positive attitude towards learning it (Grootenboer & Marshman, 2016b). However, simultaneously, Onion (2004), Nardi and Steward (2003) and Kollosche (2017) report that learners consider mathematics to be a difficult and boring subject. Further, learners cite using elementary calculation skills (Kollosche, 2017), assessment (Onion, 2004), qualifying for a well-paid job (Wiik & Vos, 2019), or "to do homework" (Alrø et al., 2009, p. 15), rather than the knowledge of mathematics, as reasons for believing that learning *secondary level* mathematics is relevant and important for them.

Kloosterman (2002) and Grootenboer and Marshman (2016b) illuminate this paradox by reporting that secondary school learners are seldom conscious of, or used to reflectively considering or questioning if and for what they need to learn mathematics, and how it is relevant and important for their lives. Based on this observation, one may ask how secondary school learners formulate their beliefs about the relevance and importance of learning mathematics for themselves. In this paper we address this research gap by exploring learners' beliefs about the relevance and importance of learning mathematics and the sources of information forming these beliefs. The research question we investigate is:

What are Norwegian secondary school learners' beliefs about the relevance and importance of learning mathematics, and what are the sources of information influencing the formation of their beliefs?

Background and context

Prior to exploring the question above, it is reasonable to enquire if secondary school learners are expected, encouraged, or trained to consider and critically reflect over their beliefs about the relevance and importance of learning mathematics for themselves. In this section, we explore this enquiry based on the socio-political discourse prevailing in the research literature and educational policy documents about the relevance and importance of learning mathematics.

Socio-political discourse on the relevance and importance of learning mathematics

On analysing learners' beliefs about the relevance of the mathematics' curriculum for themselves, Sealey and Noyes (2010) noted that learners consider mathematics "a power subject, giving access to higher paid careers and economic security" and, this "discourse (of the middle classes and their teachers) seems to go largely unchallenged" (p. 250). Grootenboer and Marshman (2016b) also pointed out that, "saying that mathematics is important is a generally unquestioned view" (p. 57). Studies such as, Ernest (2004), Pais (2013), Valero (2017) and Kollosche (2017) point at the socio-political discourse emphasising the relevance and importance of learning mathematics as a possible reason underlying these unquestioned beliefs.

The international educational policy documents issued by UNESCO (2015) and OECD (2019), and the research literature for instance, Heymann (2003) and Ernest (2005) also underpin this discourse by emphasising the usefulness of learning mathematics for learners' personal, economic, and societal development. The universal accept of this usefulness and importance of learning mathematics is further underlined in educational policies of many countries (e.g., the "Mathematics for all" policy (Allexsaht-Snider & Hart, 2001); Kunnskapsdepartementet (2019)).

The Norwegian secondary school curriculum for mathematics also reflects this socio-political discourse explicitly in its current (LK20) and former (LK06) versions. Statements like "Both boys and girls must have the opportunity to gain rich experiences from the subject of mathematics that create positive attitudes to […] the subject" (Kunnskapsdepartementet, 2006); and "Mathematics is an important subject […]" (Kunnskapsdepartementet, 2019) promote the relevance and importance of learning mathematics and aim to develop a positive attitude among learners towards learning mathematics.

However, Ernest (2004) points out that, 'There is no reason to assume that learners will regard mathematics curricula as "relevant" just because educational and political leaders do so […]' (p. 315). It is further claimed that learners' own views of mathematics and its relevance to their *personal* goals are missing from the discussion about relevancy of mathematics education, and their "beliefs about the relevance or utility of mathematics often reflects the prevailing rhetoric about the importance and high valuation of mathematics in society" (ibid., p. 316). Nonetheless, some recent studies have critically assessed this prevalent socio-political rhetoric and the roles mathematics education plays in society. These studies also outline possible outcomes these rhetoric and social roles of mathematics education may have for learners' views about the relevance and importance of learning mathematics.

Questioning the unquestioned socio-political discourse

Lundin (2012), Pais (2013), Valero (2017) and Kollosche (2018) critically evaluate the prestigious status attributed to secondary school mathematics and problematize its socio-political roles. These studies highlight that mathematics can act as a discipline training learners to be obedient, to follow rules and experience boredom in the classroom (Kollosche, 2018; Valero, 2017). Their research points at mathematics operating as a discipline to groom learners as future citizen-workers of a bureaucratic society rather than being a set of skills or knowledge relevant and important for all learners, who may have different personal goals. Therefore, these studies encourage critically questioning the dominant socio-political discourse and the emphasis laid on the relevance and importance of learning mathematics. Moreover, a call to empower learners to negotiate the subject content they learn in mathematics and its relevance for themselves has also been voiced in the research.

Mellin-Olsen (1987, 1993) emphasised that the learners should *own*[1] the aims of their education but also noted that, "the pupil has usually been considered as one who reflects on the mathematical content of the situation, and not *about* the learning situation" (1987, p. 20). Critical Mathematics Education (CME) research further addresses this concern. Alrø and Skovsmose (2002) assert the need of dialogue with the learners to, "the extent that they are able to recognise the intentions and to identify with them, they can be joint owners of their education" (p. 43). Ernest (2001) also acknowledged that empowering learners as individuals and citizens in accordance with the aims of CME will, "require the use of a questioning and decision-making learning style in the classroom. Teaching approaches should include discussions, […], the *questioning of content* and the *negotiation of shared goals*". (p. 288, italics added). This appeal underpins the need of empowering learners (mathematically, socially, and epistemologically) in

[1] That learners get the opportunities to pose and solve problems relevant and interesting for themselves, participate in the decisions about the content of their own mathematics learning, and evaluate their learning situations.

their mathematics learning processes by taking a joint ownership, questioning the relevance of the subject content, participating in decisions, negotiating shared goals, and critically evaluating their learning situation (see Ernest (2002) for details). These concerns correlate with CME's aims to empower the learners and impart democratic values and critical citizenship skills[2] in them through mathematics education.

Empowering learners to influence their learning processes along with developing their critical thinking are also the aims of learners' educational process as per the Norwegian Education Act (NEA): "The pupils […] must learn to think critically […]. They must have joint responsibility and the right to participate" (The Education Act, 1998). An understanding of this statement can be that education should empower the learners to take a joint responsibility of, think critically about, and exercise their right to co-operate and contribute to decisions concerning their education. These objectives are also echoed in the general part of the core curriculum of Norway (applicable to all the subjects taught at primary or secondary level). The 'Nordic model' of education also underlines the goals of learners' empowerment, democracy, critical citizenship through learners' education process (Andersson & Österling, 2019; Dahl & Stedøy, 2004). Thus, empowering learners to question the subject content's relevance for themselves, to negotiate shared goals, to participate in the decision-making, to reflect about and evaluate their learning situation in mathematics also become the aims of mathematics education. The statement, "Mathematics shall help pupils to […] become more aware of their own learning" listed in the Norwegian mathematics curriculum LK20 (Kunnskapsdepartementet, 2019, see section "Relevance and central values") also indicates similar concerns.

Based on the discussion above, it is reasonable to infer that secondary school learners are expected and encouraged to critically reflect over and question the relevance and importance of the subject content they learn in mathematics and get the opportunity to negotiate shared learning goals for themselves. Learners should also be encouraged to participate, take ownership of, influence and co-operate in decisions concerning their mathematics learning process. Such practice and training are seldom observed as a tradition in school mathematics (Kloosterman, 2002) despite the expectations of research and curriculum guidelines to heed learners' voices. The learners are rarely asked to think critically *about* learning mathematics (Sachdeva & Eggen, 2021) or have a say in the socio-political discourse regarding their mathematics education (Sealey & Noyes, 2010). There is little knowledge about how learners' beliefs about the relevance and importance

[2] Promoting democratic values can empower learners to participate, negotiate, co-operate, and influence the decisions concerning their own mathematics learning. Promoting critical citizenship skills can empower learners to apply their own critical judgement to the claims or declarations of oneself, any other person, interest groups or authorities. In mathematics classroom context, learners' can be empowered by involving them to participate and critically evaluate the decisions concerning their own mathematics learning. Empowered learners can in-turn promote democracy and critical citizenship in the society.

of learning mathematics are formed (Grootenboer & Marshman, 2016b), or how they can contribute to their own mathematics learning process by critically evaluating this relevance and importance against their personal goals (Ernest, 2004). Therefore, in this study we explore learners' beliefs about the relevance and importance of learning mathematics for them personally, and the sources of information influencing the formation of these beliefs. Further, we discuss implications of learners' expressed beliefs for empowering them to contribute to their own mathematics learning process.

Conceptual framework

Beliefs

The nature, structure, definition, qualities, content, influence, and classifications of beliefs is explored by different research scholars (see Green (1971); Bar-Tal (1990) and Leder et al. (2002) for instance). The significance of learners', teachers', and educators' beliefs for the teaching and learning of mathematics is explored in research literature (e.g., Goldin et al. (2009) and Maaß and Schlöglmann (2009)) examining the affective dimension of teaching or learning mathematics. However, the research is not unanimous upon a universal definition of beliefs (ibid.). Furinghetti and Pehkonen (2002) present a thorough review of research concerning definitions and characterizations of beliefs, but a discussion of all these is beyond the scope of this paper. In the following text, we present selected definitions, and classifications which can assist the aim of exploring learners' beliefs about the relevance and importance of learning mathematics and the sources of information forming them.

Definition of beliefs

Bar-Tal (1990), defines beliefs as "[…] units of cognition. They constitute the totality of an individual's knowledge including what people consider as facts, opinions, or hypotheses, as well as faith. Accordingly, any content can be the subject of a belief." (p. 12). Goldin (2002) defines beliefs as, "multiply-encoded, internal cognitive/affective configurations, to which holder attributes truth value of some kind (e.g., empirical truth, validity, or applicability)" (p. 59). Despite the difference between these two definitions, the research community seems to agree that beliefs are 'highly subjective cognitive entities about some aspect of an individual's world which are considered to be true by the individual (the subject) holding those entities'. An instance of such a cognitive entity can be a learner's belief that, "learning is to reproduce the knowledge in an exam". In this example, the learner's subjective opinion (cognitive entity/belief) is about what learning (an aspect of learner's world) is for, and this belief is held true by the learner.

Beliefs are also clearly distinguished from other notions discussed under the affective domain such as, attitudes, values, and emotions. McLeod (1989) suggested that "beliefs, attitudes and emotions differ in the ways that cognition is involved in the affective response" (p. 246). While "beliefs are mainly cognitive in nature" and are formed "slowly over a relatively long period of time", the emotions usually "have a much stronger affective component" (ibid.). Goldin (2002) defines the notion of attitudes as, "moderately stable predispositions toward ways of feeling in classes of situations", and the notion of values as, "deeply-held preferences, possibly characterized as "personal truths", stable" (p. 61). Therefore, the notions of beliefs, values, attitudes, and emotions can be listed in order of increasing affective involvement and intensity (emotions being most affective and intense, beliefs least) and decreasing cognitive involvement and stability (beliefs being most cognitive, cf. 'units of cognition', and stable, emotions least).

In this study, we follow the same distinction between beliefs, attitudes, emotions, and values as presented above. Adapting Bar-Tal's definition of beliefs, we understand learners' beliefs as their subjective opinions, hypotheses, as well as faith about the relevance and importance of learning mathematics for their lives.

Sources of information influencing learners' beliefs

Bar-Tal (1990), building on the work of Bem (1970) and Fishbein and Ajzen (1975), classified beliefs into three categories based on the sources of information influencing their formation. These three categories include *descriptive*, *inferential*, and *informational* beliefs. Descriptive beliefs are derived from the perceptions acquired through one's senses and direct subjective experiences. Inferential beliefs are formed by applying the rules of logic on beliefs (personal and/or socialized) collected in the past and making inferences based upon them. Lastly, informational beliefs are formed on the basis of information provided by outside sources such as other individuals, books, television, newspapers, etc. (Bar-Tal, 1990, p. 13).

Hernandez-Martinez and Vos (2018) acknowledge the formation of learners' relevance beliefs based on their personal motives or interest in mathematics, and/or based on a general relevance mediated to them by others[3]. Likewise, learners' importance beliefs regarding learning mathematics may be formed based on their subjective perception of values inherent to mathematics, and/or based on the importance mediated to them by others. These sources of information influencing learners' beliefs correlate with Bar-Tal's (1990) categorisation of beliefs (cf. subjective motives to descriptive beliefs and information mediated by others to informative beliefs). Therefore, we adopt Bar-Tal's (1990) classification of beliefs described above to categorise the sources of information influencing the formation of learners' beliefs (see Analyses and findings section).

[3] Social and/or political sources of information such as, teachers, elders, researchers, governmental authorities and institutions, educational policies, mass media, etc.

Relevance

Wedege (2007) suggests that the question of relevance of mathematics education is always grounded in a context involving questions such as *what* (in mathematics) is relevant and *why*. These questions are further elaborated by Nyabanyaba (1999) and Jablonka (2007) as relevance *of what*, relevance *to whom*, relevant *according to whom*, relevance *for what end* or *purpose* with the reference to both individual and general (socio-political) objectives of learning mathematics as pointed out by Wedege (2007). The term *relevance* has also been discussed with reference to words such as utility (Ernest, 2004), current or future usefulness (Sealey & Noyes, 2010), need and demand (Wedege, 2007). Therefore, the concept of relevance is not precisely defined, but is related to the notions of *usefulness* and *meaningfulness* (Hernandez-Martinez & Vos, 2018; Priniski et al., 2018). Hernandez-Martinez and Vos (2018) suggest that "usefulness is a property of the topic being learnt, […] while relevance is a *connection* between the topic being learnt, its usefulness and a learner" (p. 246). Priniski et al. (2018) consider *relevance* to be "a personally meaningful connection to the individual" (p. 12).

Learning mathematics can be useful for a learner to achieve the aim of becoming a mathematician. For another learner, learning mathematics maybe meaningful because of the joy experienced in solving mathematical tasks. In this paper, the relevance of mathematics education is understood as its property of being useful or meaningful for the learners. Further, the connections between learning mathematics and its usefulness or meaningfulness established by learners are understood as their *relevance beliefs*.

Importance

The notions of *importance* and relevance are often treated as equivalents and used synonymously or together. This synonymous relation is visible in research and different official policy documents (for instance, see Allexsaht-Snider and Hart (2001), NCTM (2000), OECD (2019) or Kunnskapsdepartementet (2019)).

Research studies in mathematics education do not explicitly differentiate relevance from importance, but this distinction is made elsewhere. Solomon and Heller (1982) assert that the difference between importance and relevance is critical, "for that which is relevant is not necessarily important, and that which is important is not necessarily relevant"[4] (p. 165). The authors explain that the importance of anything is considered based on its *intrinsic* characteristics such as meaning, import, consequences, prominence, and value, whereas the relevance of anything is considered based on its *extrinsic* characteristics such as technique, application, usefulness, and service.

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[4] As an example of this difference, learning mathematics may be relevant for a learner because of the joy experienced in solving mathematical problems, but he/she may not consider learning mathematics to be important because of the intrinsic values (e.g., power of abstraction) it promotes; or vice-versa.

In mathematics education research, the notion of relevance and learners' relevance beliefs are discussed exclusively (see Relevance section), but the importance of learning mathematics seems to be treated implicitly under its relevance. Learners' beliefs exclusively about the importance of learning mathematics (different from relevance) are seldom explored. In this study, the concepts of relevance and importance are considered as being supplementary to, but different from each other. We understand the importance of mathematics education as the import and values that are intrinsic to learning mathematics (such as power of abstraction, imagination, estimation, simulation, etc.). However, learners' beliefs about the importance of learning mathematics might diverge from this understanding of import values. For instance, research studies such as Pais (2013) and Wiik and Vos (2019) have found that learners believe learning mathematics to be important for their future because of its exchange-value in the job-market. Here, we consider learners' beliefs about the importance of learning mathematics as their *importance beliefs*.

Method

This study is part of a larger research project called Local Culture for Understanding Mathematics and Science (LOCUMS, 2016-21), aimed at exploring the role of practical tasks (rooted in learners' own interests and local culture) in the learning of mathematics and science. The data was collected under the sub-project of LOCUMS carried out in Central Norway. Two schools including learners from diverse cultural backgrounds were chosen as the sites of data collection in accordance with the research aims of LOCUMS and four classroom interventions were planned. The interventions took place in 8th and 9th grades of these two schools with learners 13-14 years of age. Each intervention included three steps of data collection. In the first step, 74 learners responded to a paper-pen questionnaire. The second step included learners working to solve practical group tasks (4-5 learners in each group) and the final step included face-to-face individual semi-structured interviews with 19 selected learners. The data analysed for this paper constitutes a part of interviews in which learners' relevance and importance beliefs about learning mathematics were explored in-depth.

A representative sample (learners having high, average, and low interest in learning mathematics) of four or five learners from each class (one learner from each group solving practical tasks) were interviewed. The selected sample included nineteen informants (11 girls, 8 boys). The interviews, each lasting for 45–90 minutes, were conducted in Norwegian, audio-recorded, and later transcribed for analyses. This paper is based upon the analyses of learners' responses to the interview questions exploring their beliefs about the relevance and importance of learning mathematics for themselves.

The interviewer was present with the interviewed learners under the interventions to assist, answer their questions and collect data while they were working on practical group tasks. Learners were therefore familiar with the interviewer and had interacted with her informally before the interviews. Interview techniques such as waiting, conforming, non-academic language, comforting the learners in case they did not answer a question, etc. were used to reduce unfortunate authority of the interviewer, and to ensure learners' honest responses. The interviewer had no teaching duties or personal relation with the learners. Written information about the research project was provided to learners' parents/guardians and their consent to interview the learners was obtained. The interviews started by informing every learner about the interviewer's duty of confidentiality and their anonymity. Learners were assured that the information provided by them would not influence their teachers, education, or grades in any way.

The choice of words in the interview questions was made to avoid a gap of understanding between the interviewer and the learners. Words such as *useful*, *need* and *use* were employed instead of the words *relevance* or *meaningful* to enquire learners' beliefs about the relevance of learning mathematics, and the words *important* and *importance* were used to enquire into their beliefs about the importance of learning mathematics. Learners were asked *if* and *why* they believe learning mathematics to be important and relevant for themselves. The interview questions included for instance, what mathematical content they find useful to learn, where they use/will use mathematics, is mathematics important for them to learn and why do they believe so, etc. They were also asked to give reasons for their beliefs to identify the sources of information underlying and forming their beliefs. The interview excerpts presented in this paper are selected due to the representativeness and clarity in learners' responses. The unit of analysis is learners' responses to the interview questions.

Exploring learners' beliefs is seen as difficult since their beliefs are subjectively valid but not necessarily static or explicit for themselves. Lester (2002) observed this difficulty and doubted if "[the data] accurately indicate what the students really believe. I do not think most students really think much about what they believe about mathematics and as a result are not very aware of their beliefs" (p. 353). Therefore, capturing learners' beliefs accurately is difficult, which results in the limitation that the findings of this study cannot be generalized and will only represent 'there and then' beliefs of selected Norwegian learners. We dealt with this limitation to some extent by asking the learners to not only express their beliefs, but also to mention the reasons for holding them. The credibility of the findings of this study is further established by discussing these findings with previous research results (see Discussion).

Analyses and findings

The analyses were initially driven by the data where learners' answers including words such as *useful*, *fun*, *need* and *use* were taken as indications of their relevance beliefs, and their use of words such as *important* and *importance* were taken as indications of their importance beliefs. Later, Bar-Tal's (1990) categories of beliefs were used to classify learners' beliefs as descriptive, inferential or informational based on the sources of information forming these beliefs. The abductive process of going back and forth between the data and conceptual framework including the notions of relevance, importance and categories of beliefs resulted in the following schematic overview table (Table 1) used to categorise learners' beliefs.

Beliefs' category ⟶ **Relevance and Importance beliefs about learning mathematics** ↓	Descriptive	Inferential	Informational
Relevance beliefs *[Extrinsic]*			
Importance beliefs *[Intrinsic]*			

Table 1 Relevance and importance beliefs table (before data analysis)

After the data analyses process, the table above was filled with learners' beliefs about the relevance and importance of learning mathematics expressed during the interviews (see Table 1 in attachment). In the next two sections, we present and analyse learners' interview responses on relevance (useful/meaningful) and importance of learning mathematics for themselves.

The relevant mathematics

Fifteen out of nineteen learners mentioned elementary arithmetic as the most relevant mathematical subject matter because of its usefulness in their everyday life, for instance in shopping and budget estimation. Only four learners gave examples of secondary-level mathematical subject content they use in their lives such as using maps and compass for spatial orientation, calculating mass, density,[5] and knowledge of geometrical construction in case of becoming a carpenter. Some replies also included references to assessment in mathematics, as shown in the following excerpt:

[5] This learner (L_8) mentioned that one can for instance make an electric bicycle out of things one has in the garage if one can calculate mass, density etc.

I[6]: What is useful for you to learn in mathematics?
L_{15}[7]: I find most of it quite useful … eh … well, useful … almost everything, especially just plus and minus and division and … multiplication … these are things I find very useful …
I: But what about other stuff you learn now, like algebra and x, y and z or …
L_{15}:Ok … I do not know how useful it will be to me, but I think it is fun … [laughing]
I: [Also laughing] Yes, you find it fun, but …
L_{15}:I do not know how useful it will be in the future … I have not experienced a situation where I would need it except for the tests … so …

The learner first mentions *most of* the mathematical subject content as quite useful. Saying the word useful once again, "eh … well, useful …", s/he goes on to include *almost everything*. However, to give specific examples s/he mentions basic arithmetic skills *especially* to be *very useful*.

When the learner is specifically asked about the usefulness of learning algebra s/he replies that though s/he finds learning algebra to be *fun*, s/he *does not know* how useful it will be in the future and specifies that s/he has *not experienced a situation* where s/he would need algebra *except for the tests*. The learner's reply indicates that though s/he does not know the relevance of learning algebra for future life, still assessment and evaluation criteria in mathematics serve as reasons for believing that it is relevant.

This belief in the usefulness of basic arithmetic seems to be based on direct experiences of needing these skills to do everyday calculations and is categorised as a *descriptive* belief. Whereas the reply regarding algebra indicates that learning algebra is experienced to be meaningful (*fun*), and the belief about its usefulness is grounded in the inference that s/he will need it for passing the tests. Therefore, it is categorised as an *inferential* belief (see Table 1 in attachment).

Using the phrase, "I do not know …" twice in the answer indicates that the learner may not have thought much about the relevance of learning mathematics for her/himself. Some other learners also expressed the same:

I: What do you yourself believe to be useful to learn in mathematics?
L_{12}: To calculate …
I: OK … and?
L_{12}:Ah … [long pause] … I do not know …
I: Have you ever thought about it?
L_{12}:No … I just think that in our lessons … that, OK, now we are going to calculate and solve problems in our book … I do not think more beyond that …

The learner mentions that learning *to calculate* in mathematics is useful. On being asked to mention more examples, s/he did not come up with other useful subject matter to learn in mathematics (saying "I do not know" after a long pause). When asked if s/he has *ever thought about* what is useful to learn in mathematics, the

[6] *I* stands for the *interviewer.*
[7] L_n specifies the *learner's* reply in n^{th} interview.

learner answered that s/he has *not* done so. S/he added that in mathematics lessons s/he thinks that they will *calculate and solve problems in their book,* and s/he does *not think more beyond that*. This remark indicates that learners are not used to consciously reflect over the relevance of learning mathematics for their lives and such reflections may not be a part of their "routine" (*calculate and solve problems in our book*) mathematics classroom practices.

While some learners did not seem to be thinking over the relevance of learning mathematics for themselves, other learners said that they have heard from *others* that learning mathematics is relevant for them. The following extract exemplifies this:

> I: Why are you going to need it [algebra] …?
> L_5: I have just heard that one needs it …
> I: Heard? Where?
> L_5: From teachers and other adults …
> I: But have you ever asked them where they use it?
> L_5: No …
> […]
> I: Has he [the teacher] said anything about where you will use an equation?
> L_5: No …
> I: Ok … well … but you still think that though you do not know where one needs or can use it … yet you think that it is useful to learn?
> L_5: Yes … because we learn it …

On being asked *why* s/he is going to need algebra and equations; the reply was that s/he has *just heard that one needs it* from her/his *teachers and other adults*. This reply indicates that the learner accepts the stated requirement of learning mathematics. The source of the learner's belief in the relevance of learning mathematics is information received by others and therefore this belief is categorised as an *informational* belief. When asked if s/he has asked the teacher and other adults about where they use algebra or where s/he can use equations, the reply was that s/he has not questioned this claim. The reasons for not questioning such claims are not clear but the last line in the interview segment, "yes … because we learn it" indicates that this learner trusts his/her school system and that the curricula chosen for her/him must be relevant. This trust is also reflected in the following example:

> I: But can you use equations and algebra somewhere [later in your life] …?
> L_{11}:Eh … yeah, not that I know where one will use it but I know that one can use most of what we learn in mathematics somewhere but there are more important things that we should have really learnt first such as … how one pays a bill and how one … eh … everything like that … eh … I have not learnt much of that … though we will definitely learn such stuff later …

This learner did *not know where one will use* the algebra and equations they learn, but s/he knows that *one can use most of what they learn in mathematics* *somewhere*. However, s/he also reflects critically and admits that there are *more important things* that they *should have really learnt first such as…how one pays a bill …* and which they *have not learnt much of* (e.g., using mathematical knowledge in their adult life), yet simultaneously expresses her/his trust in the education system that they will *definitely* learn that stuff later. This trust is more visible in leaners' replies about the importance of learning mathematics, presented in the following section.

The important mathematics

Despite struggling (responses like, *I do not know*, long pauses, *I have just heard that one needs it*, etc.) to find the usefulness of learning secondary-level mathematics in their lives, *all* the learners answered that learning mathematics is important for their lives. This importance of learning mathematics surfaced in several interviews (in form of various reasons) and inspired us to explore *why* the learners considered learning 8th and 9th grade mathematics to be important. The following interview segments present some responses:

> I: As you said that it is difficult to learn but do you think it will be important for your life later?
> L_1: Much of it is important … yes … like … [long pause]
> […]
> I: So, why do you think that really? What can you use it for later in your life then?
> L_1: It is about what you will work with … like … yeah so, if one will work in a shop or be a hairdresser and for example if one has to stand at the cash counter, so, it's a must that one has enough knowledge of mathematics, one should be able to do that … and much like that for many different jobs one must have a knowledge of mathematics …
> […]
> I: But can't one find any jobs where one does not need to know much about mathematics and science?
> L_1: There are no such jobs…or I do not think one can find any such jobs…
> L_1 expressed earlier in the interview that s/he finds learning mathematics and science to be difficult since these subjects require thinking hard (much concentration), yet on being asked about the importance of learning mathematics s/he answered, "much of it is important". When s/he tried to come up with instances of this important subject content s/he did not mention any (tries to recall while saying "yes … like …" but takes a long pause). On being asked about the reason of thinking mathematics to be important, s/he mentioned future job opportunities (though being a hairdresser or handling a cash counter rarely require more than basic arithmetic skills). The learner even inferred that there do not exist any jobs not requiring the knowledge of school mathematics and science.

Another reason for considering learning mathematics as important for future, included helping one's own children in their mathematics homework when it is their turn:

I: Do you think that what you learn in mathematics and science will be important for you, later in your life?
L_2: Yeah … they say that at least …
I: Who?
L_2: The teachers … they say that at least …
I: Yeah … but do you think so yourself? You should answer for yourself now …
L_2: Yeah … I maybe think that too … like if you will have children then you can help them with their homework …
I: Yeah … but like in everyday life like … in real situations … where are you going to see mathematics and science actually?
L_2: Eh … it is difficult! [pause]
I: Where are you going to use it in your life?
L_2: Where I would use … no … I do not know …
[…]
I: You do not know …
L_2: No … I have not thought much about it …

Being asked about the importance of learning mathematics, L_2 instantly replied, "yeah…they say that at least", and when asked to specify who, s/he quickly says that it is "the teachers". The influence of what teachers (classroom's authority) communicate about the importance of learning mathematics becomes clear from this example. On insisting to answer the question based on her/his personal thoughts and experiences, L_2 replied, "I maybe think that too". The learner argued for her/his answer by imagining a future situation where s/he can use her/his knowledge in mathematics to help her/his children in their homework. L_2 found it *difficult* to find instances where s/he would use advanced mathematics in her/his daily life. The statement, "No … I have not thought much about it", indicates that the learner does not often reflect on the reasons for believing in the importance of learning mathematics. L_2's belief in the *importance* of learning advanced mathematics indicates that the socio-political importance of learning school mathematics can get reinforced among learners since they 'hear' about it from their teachers and do not reflect over or critically question such claims.

In several interviews, learners stated that they do not think about the importance of learning mathematics since they are informed about its importance by their teachers, parents, popular media etc. Interview segment with L_5 illustrates this case even more:

I: And important?
L_5: It is not the most important thing but … like … eh … I just mean that people should know about it and be able to do it, but I do not know why … we learn it at school so one thinks that one will use it …

This learner explicitly expresses her/his trust in the schooling system that even though learning mathematics may not be *the most important thing*, *people should know* the mathematics taught in the school (though admitting, *but I do not know why* …) since they *learn it at school* (a socio-political institution). However, the same trust may make the learner disclaim her/his right (as per NEA) to critically question the importance and relevance of what they are taught.

Learner's replies in the interview segments presented above indicate their *inferential* (e.g., inferring that they will need mathematics to teach their own children, or in their future jobs, etc.) and *informational* (e.g., the teacher/other has told them about the importance of learning mathematics) importance beliefs about learning mathematics. However, none of the learners' responses included their *descriptive* (directly experienced) beliefs about the importance of learning mathematics, based on the intrinsic values (e.g., the power of abstraction, estimation, etc.) of learning mathematics (see Table 1 in attachment).

Discussion

The interview responses of learners indicate that a distinction between the notions of relevance and importance, suggested by Solomon and Heller (1982), can also be made in the context of learning and teaching mathematics. This distinction may not be clear for the learners, but it can be noticed in learners' replies expressing their relevance beliefs and importance beliefs about learning mathematics. Their responses to the questions regarding *relevance* of learning mathematics are based on the extrinsic application or usefulness (*use-value*) of mathematics. However, their answers about the *importance* of learning mathematics point towards the inferred or informed prominence or consequences (*exchange-value*, in terms of job, degree, own children's education, etc.) of learning mathematics, rather than the intrinsic meaning or importance of mathematical knowledge itself. The choice of words in the interview questions can be a partial reason, but this dissimilarity was consistently noticed in the responses of many learners.

The relevance of learning mathematics surfaced in learners' responses both as its usefulness for calculations (L_{12} and L_{15}) and meaningfulness for enjoyment (*fun*, L_{15}). Learners' responses as presented in 'The *relevant* mathematics' section, exhibit their strong belief in the relevance of learning 8^{th} and 9^{th} grade school mathematics despite referring mostly to elementary arithmetic skills as the subject content they use in their daily lives. A majority of secondary school learners interviewed by Kollosche (2017) also associated the relevance of learning mathematics to the mastery of basic calculation skills. L_{15}'s response about needing mathematics in the tests is coherent with the findings of Onion (2004) and Alrø et al. (2009) where learners reported that the mathematics they learn is useful only in their mathematics lessons, to do homework, and for exams. Learners' relevance beliefs about learning mathematics further seem to be formed on the bases of all the three sources of information suggested by Bar-Tal (1990), that is, their descriptive, inferential and informational beliefs.

The importance of learning mathematics becomes apparent in learners' reference to future situations in which they expect to require mathematics, such as to get a job (L_1) or to teach mathematics to their own children (L_2), which can be seen as a circular argument for learning mathematics. This finding correlates with Wiik and Vos (2019), reporting that learners choose to learn mathematics in anticipation of getting high-paid jobs in future. The analyses of interview extracts presented in 'The *important* mathematics' section reveal learners' strong belief in the importance of learning mathematics for their lives despite not being able to justify this belief. Such importance beliefs seem to support the suspicion of Kollosche (2018) and Valero (2017) that mathematics as a school discipline is required to train learners to become future citizen-workers instead of critical citizens. Further, learners' importance beliefs about learning mathematics seem to be formed on the bases of inferences they make or the information they receive from others, instead of their own direct experiences.

The number of learners replying, "I do not know" and "I have not thought about it" (15 of 19 learners), and the frequency of such replies indicate that reflecting consciously and critically over the relevance or the importance of learning mathematics for their lives is not usual for them, as Kloosterman (2002) also pointed out. Such reflections do not seem to be a part of their mathematics classroom routine or expectations (L_{12}) either. They express their trust in the educational system, and state that the content chosen by the authorities (teachers, other adults, school) and policy makers for them to learn should be relevant and important for them. Teachers' and other adults' statements also seem to influence the formation of their beliefs about the relevance and importance of learning secondary-level mathematics. The socio-political status and value attributed to mathematical knowledge for succeeding in life plays a significant role in forming learners' beliefs. Similar to the findings of Sealey and Noyes (2010) and Grootenboer and Marshman (2016b), none of the 19 interviewed learners questioned or critically evaluated this dominant socio-political rhetoric of highly emphasising the relevance, importance and requirement of learning mathematics.

This unquestioned acceptance contrasts with NEA's and CME's intention to empower learners to think critically, co-operate in decisions concerning their own (mathematics) educational processes and develop critical citizenship skills through learning mathematics (Ernest, 2001; Mellin-Olsen, 1987). Consequently, a *tension* emerges between the intentions of NEA and CME, and the expressed realization of these aims in learners' interview responses, bearing consequences for empowering learners to contribute to their own mathematics learning process by thinking critically.

Learners (like L_{11}) exhibit the potential of critical consideration and suggesting practical mathematical content (paying bills) that could be taught before algebra or equations, still such suggestions may not be further conveyed to their teachers or higher authorities under the presumption that such content will be chosen and taught later. L_{11}'s answer indicates that given the opportunity to participate and cooperate in the decision-making process, learners can act as discussion partners suggesting alternative mathematical subject matter personally relevant and important for them. However, in these interviews, only a few learners make such suggestions. It is also unknown if their proposals get forwarded or have an influence on their mathematics learning content or activities. Consequently, their potential for participating in and critically questioning the dominant socio-political rhetoric of relevance and importance of learning mathematics may remain hidden, and their empowerment and critical citizenship skills unpolished.

The data analysed in this paper does not uncover the cause of this tension, but a possible reason may be a gap in the understanding of becoming an *empowered* learner through learning mathematics. By this gap, we mean closely linking the notion of learners' empowerment to becoming what Ernest (2002) terms as a *mathematically*[8] empowered learner, but *not* a *socially* and *epistemologically* empowered learner[9]. The formulations using the terms 'critical', 'citizenship' and 'democracy' in LK06 can be seen as exemplifying this gap of understanding[10]. We argue that adopting a broader understanding of becoming an empowered learner through learning mathematics suggested by Ernest (2002) may contribute to realize NEA's and CME's aim of learners' empowerment and critical citizenship, as envisioned in the 'Nordic model' of (mathematics) education (Dahl & Stedøy, 2004).

Implications

The distinction in learners' replies concerning the relevance and importance of learning mathematics implies that the concepts of *relevance* and *importance* deserve to be treated distinctively, rather than synonymously, in mathematics education research. This distinction can contribute to differentiate and communicate the relevance (extrinsic values) and importance (intrinsic values) of teaching and learning mathematics in general. Specifically, this difference can contribute to understanding nuances underlying learners' (also educators') beliefs about the relevance and importance of teaching and learning mathematics. These nuances can help: (a.) exploring the motivation and reasons of learners' (also educators') involvement in the learning and teaching of mathematics more precisely; and (b.) posing and answering further research questions (e.g., how can the learners directly experience the intrinsic values of learning mathematics, how does an understanding of intrinsic values of learning mathematics affect learners' mathematics learning process, etc.).

[8] Ernest (2002) suggests that a mathematically empowered learner can use his/her mathematical knowledge to pose, solve, evaluate, and discuss mathematical problems and models critically.

[9] A socially and epistemologically empowered learner, as per Ernest (2002) can critically analyse and challenge the underlying socio-political assumptions, authority, power structures, roles, uses, abuses, purpose, etc. associated with learning mathematics.

[10] In the current and revised version of the Norwegian mathematics curriculum, LK20, the scope of reference to the terms critical, democracy and citizenship is expanded. However, democracy and citizenship are still closely connected to data quantification and its critical evaluation rather than empowering learners to evaluating one's own learning situation while learning mathematics critically.

The 8th and 9th grade learners interviewed in this study will be making crucial decisions regarding their own future mathematics education and career during their tenth grade. However, the analyses indicate that these learners are not used to making reflected choices and critically evaluating the decisions about their own mathematics learning process. Involving learners in decisions regarding their own mathematics learning process can be a step towards mathematics education for learners' empowerment and critical citizenship. Therefore, mathematics classroom practices can reflect the aim of promoting learners' empowerment, for instance by incorporating a questioning learning style, negotiating shared goals, critically evaluating the subject content in mathematics, etc. to empower the learners to take informed decisions and apply critical judgements to their own mathematics education.

The findings of this study also bear implications for the formulations employed in the mathematics curriculum of Norway. The references made to the notions of critical thinking, democracy and citizenship can invite and encourage mathematics educators to incorporate not only mathematical, but also social and epistemological empowerment in mathematics teaching and learning practices.

Conclusion

This study contributes to the research field by establishing that there exist nuanced differences in learners' beliefs concerning the relevance and importance of learning mathematics, and that this distinction should be acknowledged in mathematics education research. Mapping of sources influencing learners' belief formation as descriptive, inferential, and informational, contribute to an increased understanding about the sources of information that influence the formation of learner's beliefs about the relevance and importance of learning mathematics for their own lives.

We also maintain that though the interviewed learners are not used to reflecting over the relevance and importance of learning mathematics, some of them exhibit a potential to suggest constructive alternatives to improve their own mathematics learning process. Giving learners time and inviting them to critically evaluate and co-operate in decisions concerning their own mathematics learning activities can encourage them to voice their suggestions. Making these changes in mathematics learning and teaching can assist in realizing NEA's and CME's aim of mathematics education for learners' empowerment and critical citizenship.

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APPENDIX

Relevance and importance:

As discussed in the article, relevance and importance of learning mathematics are understood as two different concepts. Relevance is understood as the extrinsic qualities of learning mathematics related to its application, usefulness, techniques, and services mathematical knowledge can provide to organize the society or solve technical issues. Importance of learning mathematics is, on the other hand, understood as the intrinsic values, meaning and import inherent to mathematics and mathematical knowledge, for instance, the power of abstraction, simulation, imagination, estimation, and optimization, which can be learnt by studying mathematics.

Beliefs

While analysing learners' interview responses, it was observed that their beliefs can be descriptive, inferential, or informational depending upon the sources of information forming these beliefs. These three categories of beliefs are explained by Bar-Tal (1990) as:

1. **Descriptive beliefs:** formed based on one's own direct experiences.
2. **Inferential beliefs:** formed based on rules of logical inferences.
3. **Informational beliefs:** formed based on information gathered through other sources.

Both descriptive, inferential, and informational beliefs can be about the relevance of learning mathematics or about the importance of learning mathematics. Therefore, these learners' beliefs can be further classified as following:

1. *Descriptive relevance beliefs*: learners' beliefs about the relevance of learning mathematics based on their direct experiences.
2. *Inferential relevance beliefs*: learners' beliefs about the relevance of learning mathematics based on their logical inferences.
3. *Informational relevance beliefs*: learners' beliefs about the relevance of learning mathematics based on the information they receive from other sources.
4. *Descriptive importance beliefs*: learners' beliefs about the importance of learning mathematics based on their direct experiences.
5. *Inferential importance beliefs*: learners' beliefs about the importance of learning mathematics based on their logical inferences.
6. *Informational importance beliefs*: learners' beliefs about the importance of learning mathematics based on the information they receive from other sources.

Relevance and importance beliefs table:

The following table resulted after data analyses and clarifies how the conceptual framework is used while analysing the data. Examples of learners' statements and how they were identified using the categories of beliefs mentioned over are shown below:

Beliefs' category → **Relevance and Importance beliefs about learning mathematics** ↓	Descriptive	Inferential	Informational
Relevance beliefs *[Extrinsic]* (Technique, Application, Service, Usefulness)	Use it for shopping, calculations, budget, etc. Use it for daily arithmetic, scaling for recipes, finding out volume, area, etc.	Use it for passing tests. Use it to get into higher studies.	Will use it because the teacher says so.
Importance beliefs *[Intrinsic]* (Meaning, Import, Value, Prominence, Consequences)	Learners' responses in the interviews do *not* include clear examples which point at the intrinsic values of learning mathematics, for instance, the ability of abstract thinking, making estimations, etc.	Important because I will need it for my children to help them in their homework. Important to get a job.	Important because it is taught at the school. Important because we learn it at school so one thinks that one will use it. *(Because mathematics has a value and prominent place and status in society)*

Table 2 Relevance and importance beliefs table (after data analysis).

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